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GUIDEPOST FOR TECHNICAL READERS SEPTEMBER, 1949, *PROCEEDINGS*

Items of value or significance to readers in various fields are here listed. For convenience the distribution is given in terms of the Society's Technical Divisions

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(A constant effort is made to supply technical material to Society members, over the entire range of possible interest. In so far as your specialty may be covered inadequately in the foregoing list, this fact is a gage of the need for your help toward improvement.—Ed.)

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

STREAM FLOW VARIABILITY

By E. W. Lane,[1] M. ASCE, and Kai Lei[2]

Synopsis

An index of stream flow variability is suggested, in the form of the standard deviation of the logarithm of the stream discharge. The method of determining the index is described and the values from the flow records of a large number of streams in the eastern part of the United States were obtained. The possibilities of using this index in estimating the duration curve of the stream flow, where flow records are short or nonexistent, is indicated.

A study of the indexes, together with the conditions of the respective watersheds, demonstrated that in the northeastern quarter of the United States the geology (including soil cover) and the presence of lakes and swamps are the most important factors in stream flow variability during the greater part of the time and that the other factors have relatively smaller effect. This paper is a condensation of a more detailed paper, a copy of which is deposited in the Engineering Societies Library[2a] where it may be examined, or from which copies or microfilms may be secured.

The Purpose of a Stream Flow Variability Index

The variability of the flow of a stream has long been recognized as an important factor in the suitability of the stream for various purposes, such as water power and water supply. Hydrologists have recognized that the flow in certain streams varied greatly and that in others it was very uniform, but no generally accepted method has been evolved of indicating the degree of uniformity or of variability in a quantitative way, so that two streams could be compared with each other with respect to this characteristic. It is believed that there is need of such an index, and that if a satisfactory one is developed it will be found to have considerable usefulness in analyzing hydraulic engineering projects, especially in countries where stream flow records are meager.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **February 1, 1950.**

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By reducing the variability of flow to a single number it is possible, from a comparison of the values obtained for a large number of streams, to study the influence of the various factors such as geology, soil cover, lakes and swamps, steepness of topography, position of streams with respect to synchronization of peaks, and vegetation and rainfall, and thus to clarify greatly the fundamental relations of stream flow. It is also possible to produce synthetic duration curves with few data, which will be useful in studies of hydraulic engineering projects.

Duration Curves of Stream Discharge

Since the duration curve of stream discharge showed the extent of the variation of flow of the stream, it appeared probable that the best index of flow variability would be based upon this curve. The usual form of duration curve of stream discharge is that shown in Fig. 1, in which is plotted, on ordinary

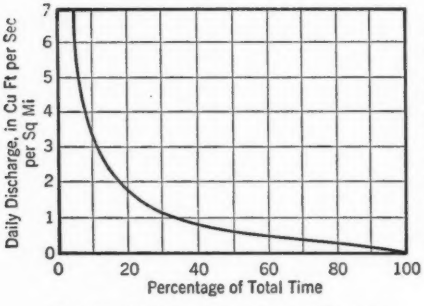

Fig. 1.—Duration Curve of Stream Discharge

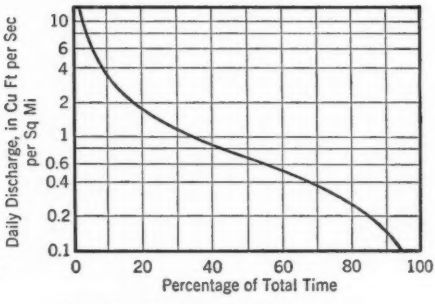

Fig. 2.—Logarithmic Plotting of a Duration Curve

coordinates, a curve showing the percentage of the total time covered by the flow record during which the stream discharge was less (or greater) than the various discharge values. This form of curve appears to have been first developed in the United States in the late 1880's or early 1890's and has been extensively used in water power and other studies.[3,4]

When ordinary coordinates are used for duration curves as in Fig. 1, in order to fit them on a sheet of reasonable size, the discharge scale is usually such that the low flows cannot be read accurately. This can be remedied by plotting the discharge on a logarithmic scale, as shown in Fig. 2. A still better presentation has been suggested by the late Allen Hazen,[5] M. ASCE, who plotted the curves on his logarithmic probability paper and secured a curve which, for the greater part of the total time, closely followed a straight line, as shown in Fig. 3. It departs from the straight line at the ends, but for only a small part of the total time, although, because of the distortion of the scale in the probability plotting, this small part of the time covers a large proportion of the space on the graph.

[3] "Duration Curves," by H. Alden Foster, *Transactions*, ASCE, Vol. 99, 1934, p. 1213.

[4] "Application of Duration Curves to Hydro-Electric Studies," by G. H. Hickox and G. O. Wessenauer *ibid.*, Vol. 98, 1933, p. 1276.

[5] "Power Estimates, from Stream Flow and Rainfall Data," by Dana Wood, *Journal*, Boston Soc. of Civ. Engrs., March, 1916, p. 77, and June, 1916, p. 299.

A stream with very uniform flow will produce a curve with little slope, as shown by curve A, Fig. 3, and a variable stream will give rise to a curve of considerable slope, as shown by curve B. The steepness of the slope can be used as a measure or index of the flow variability.

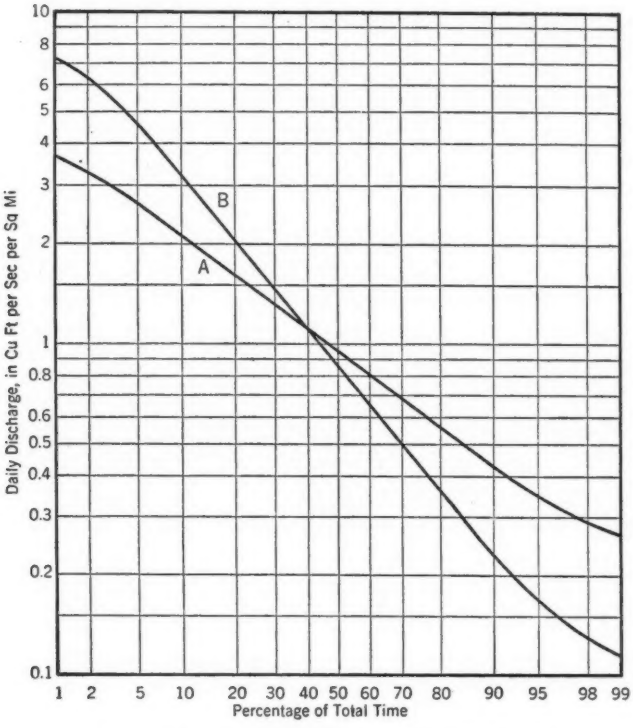

FIG. 3.—PROBABILITY PLOTTING OF A DURATION CURVE

PREVIOUS STUDIES OF DURATION CURVE PROPERTIES

Dana M. Wood,[5,6] M. ASCE, seems to have been the first to suggest that the flow characteristics of streams could be studied by comparing their duration curves. Other studies in this field were made by H. K. Barrows,[7] M. ASCE, Thorndike Saville and J. D. Watson,[8] Members, ASCE, and J. H. Morgan.[9] To a large extent the writer's idea was obtained from the work of W. C. Krumbein[10] and G. H. Otto[11] in the field of sediment particle analysis.

[6] *Transactions*, ASCE, Vol. LXXXV, 1922, p. 106.

[7] "Water Power Engineering," by H. K. Barrows, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1927, p. 128.

[8] "An Investigation of the Flow-Duration Characteristics of North Carolina Streams," by Thorndike Saville and John Dargan Watson, *Transactions*, Am. Geophysical Union, Vol. 14, 1933, p. 406.

[9] "Flow-Duration Characteristics of Illinois Streams," by J. H. Morgan, *ibid.*, Vol. 17, 1936, p. 418.

[10] "Manual of Sedimentary Petrography," by W. C. Krumbein and E. J. Pettijohn, D. Appleton-Century Co., Inc., New York and London, 1938.

[11] "A Modified Logarithmic Probability Graph for the Interpretation of Mechanical Analysis of Sediments," by G. H. Otto, *Journal of Sedimentary Petrology*, August, 1939, p. 62.

These men applied statistical methods to the analyses of particle size data and obtained an index of size variation which has been widely adopted in the field of sediment studies. The writers have used the same procedure and have adopted the standard deviation of the logarithm of the discharge as the index of stream flow variability.

The application of statistical methods to the analysis of hydrologic data has been extensively studied by Mr. Hazen,[12,13] the late L. Standish Hall,[14] M. ASCE, H. Alden Foster,[15] M. ASCE, R. D. Goodrich,[16] M. ASCE, J. J. Slade, Jr.,[17] and L. R. Beard,[18] Jun. ASCE. Although many valuable ideas were revealed by these studies, the development of an index of stream flow variability does not appear to have been considered. The logarithmic probability paper developed by Mr. Hazen in his first paper[12] was used extensively in this study, however.

To discuss both the statistical theory underlying the stream flow variability index herein proposed and also its application to hydraulic problems would extend this paper beyond a reasonable length. Those interested in the statistical theory will find it treated at length in the papers cited herein as well as in numerous texts on statistics. In the development of this paper various phases of the mathematical theory were studied but as they proved to be unimportant to the main theme of the paper they have been omitted.

For those who are not familiar with statistical theory, it may be explained that the standard deviation is the index used by the statistician to indicate the magnitude of the variation of a quantity that fluctuates. Where the value of the index is high, the variation is great; and, where the index is low, the variation is small. Many civil engineers are familiar with this from its application in triangulation and base line measurement, where the standard deviation is known as the "root mean square error."

Determining Variability Index of Streams in the United States

To develop the possibilities of an index of stream flow variability all available data on duration curves of streams in the United States east of the 100° meridian were studied. This investigation included more than 220 discharge measuring stations but they are located mostly in the northern portion of this section. These data were drawn from a large number of sources, mostly from reports by various states on the flow of their steams prepared in cooperation with the Geological Survey (U.S.G.S.) of the United States Department of the Interior. The duration curve data were based on mean daily flow. Only records of 10-year duration or more were used, and only streams on which artificial control was absent, or not an important factor, were studied.

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[12] "Storage to be Provided in Impounding Reservoirs for Municipal Water Supply," by Allen Hazen, *Transactions*, ASCE, Vol. LXXVII, December, 1914, p. 1539.

[13] "Flood Flows," by Allen Hazen, John Wiley & Sons, Inc., New York, N. Y., 1930.

[14] "The Probable Variations in Yearly Run-Off As Determined from a Study of California Streams," by L. Standish Hall, *Transactions*, ASCE, Vol. LXXXIV, 1921, p. 191.

[15] "Theoretical Frequency Curves and Their Application to Engineering Problems," by H. Alden Foster, *ibid.*, Vol. LXXXVII, 1924, p. 142.

[16] "Straight Line Plotting of Skew Frequency Data," by R. D. Goodrich, *ibid.*, Vol. 91, 1927, p. 1.

[17] "An Asymmetric Probability Function," by J. J. Slade, Jr., *ibid.*, Vol. 101, 1936, p. 35.

[18] "Statistical Analysis in Hydrology," by L. R. Beard, *ibid.*, Vol. 108, 1943, p. 1110.

In the early part of these studies, graphical methods were used entirely; but it was found that, for curves giving a high index, it was not possible, by eye, to make a determination that was certainly free from considerable effects introduced by variations in personal judgment. In order to have all the data on a reasonably accurate and uniform basis, a numerical method was used. From the duration curve plotted the values of discharge at 10% intervals from 5% to 95% of the time were read off. The logarithms of these discharges were then found and the standard deviation of these logarithms (which is the writers' variability index) was then computed by (1) taking the mean of these logarithms, (2) computing the difference of each of the ten logarithms from this mean, (3) squaring these differences, (4) taking the sum of the squares, (5) dividing it by 9, and (6) taking the square root of this value.

Factors Affecting the Variability Index

In 1936, J. H. Morgan[9] enumerated seven factors that affect the distribution of stream flow—topography, arrangement of tributaries with regard to time of concentration of surface flow, geologic structure, soil, vegetation, weather, and human developments related to flow of water. It is believed that these factors are the main ones controlling the flow, but some of them have been found to be more important than others. Since streams on which human developments have materially affected stream flow have not been included in this study, this factor does not enter. In the discussions of the results obtained in this paper, geologic structure and soils will rarely be considered separately, but will both be included under the term "geology."

The influence of these factors, as discussed in this paper, refers to the flow of the streams under the ordinary conditions and includes the effects for the greater proportion of the time. These effects may or may not be the same as the effects of the same factors on the flow of streams during large floods or in extreme droughts which usually occur less than 1% of the total time.

Results of Variability Index Determinations

The results of the variability index determinations for the 224 streams investigated, together with data on the drainage area, years of record, mean discharge, and source of the duration curve data, are presented in Table 1. These results can best be discussed by dividing them into the states in which the gaging station on the stream is located.

Variability indexes were obtained for 25 stations located in Iowa (Table 1(*a*)). The values of the variability index for the streams located principally in Iowa varied from 0.94 to 0.29. The land drained is usually rolling, without great differences in elevation, and is mostly underlain with sedimentary rocks. There are few lakes. One of the surprising results obtained was the low variability found in the regions of greatest relief. This is believed to be explained by the storage capacity available in the deeply incised porous strata of this region.

The results of the thirty-nine determinations in Wisconsin (Table 1(*b*)) follow a definite pattern. The streams in the relatively rugged region in the southwestern corner of the state gave low values of from 0.15 to 0.26, similar

TABLE 1.—VARIABILITY INDEX DETERMINATIONS FOR 224 STREAMS

Station No.	River	Station	Drainage area (sq miles)	Period of record	Flow variability index	Mean flow (cu ft per sec per sq mile)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
		(*a*) STREAMS IN IOWA				
1	Big Sioux	Akron	8,891	1935–1943	0.65	0.077
2	Nishnabotna	Hamburg[a]	2,800	1929–1942	0.58	0.238
3	Des Moines	Boone	5,490	1934–1943	0.64	0.278
4	Raccoon	Van Meter[a]	3,410	1933–1943	0.58	0.218
5	Des Moines	Des Moines	6,818	1933–1942	0.63	0.267
6	Des Moines	Ottumwa	13,200	1931–1940	0.59	0.246
7	Des Moines	Keosauqua[b]	13,900	1903–1927	0.50	0.382
8	Mississippi	Keokuk	119,000	....	0.25	....
9	Skunk	Ames	320	1934–1943	0.94	0.350
10	Skunk	Coppock	2,890	1913–1928	0.53	0.495
11	Skunk	Augusta	4,290	1915–1928	0.54	0.502
12	Iowa	Marshalltown[c]	1,500	1915–1927	0.51	0.471
13	Iowa	Iowa City[c]	3,230	1908–1927	0.49	0.457
14	Iowa	Wapello[c]	12,480	1915–1927	0.38	0.489
15	Cedar	Mitchell	845	1934–1942	0.39	0.341
16	Lime Creek	Mason City	535	1934–1943	0.63	0.343
17	Shellrock	Clarksville[c]	1,700	1915–1927	0.46	0.363
18	Cedar	Janesville[c]	1,660	1915–1927	0.38	0.407
19	Cedar	Cedar Rapids[d]	6,640	1903–1940	0.36	0.442
20	Wapsipinicon	Independence	1,060	1934–1943	0.64	0.442
21	Wapsipinicon	Dewitt	2,300	1935–1944	0.45	0.564
22	Yellow	Ion	224	1935–1944	0.35	0.553
23	Turkey	Garber	1,530	1934–1943	0.44	0.526
24	Little Maquoketa	Durango	130	1935–1944	0.39	0.579
25	Maquoketa	Maquoketa	1,550	1913–1933	0.29	0.606
		(*b*) STREAMS IN WISCONSIN				
1	St. Croix	Danbury[e]	1,550	1915–1942	0.19	0.761
2	St. Croix	Grantsburg[e]	2,820	1926–1942	0.24	0.705
3	Namakagon	Trego[e]	433	1915–1927	0.14	0.933
4	St. Croix	Rush City[e]	5,120	1926–1942	0.31	0.583
5	St. Croix	St. Croix Falls[e]	5,930	1911–1942	0.30	0.566
6	Apple	Somerset[e]	550	1915–1942	0.24	0.471
7	Jump	Sheldon[e]	510	1916–1942	0.62	1.040
8	Tomahawk	Bradley[e]	422	1915–1927	0.20	1.014
9	Pine	Power Plant[e]	543	1924–1942	0.32	0.807
10	Menominee	Twin Falls[f]	1,790	1914–1929	0.27	1.002
11	Red Cedar	Menominie[e]	1,760	1914–1942	0.25	0.678
12	Chippewa	Durand[e]	9,010	1929–1942	0.29	0.879
13	Eau Claire	Augusta[e]	500	1915–1926	0.53	0.728
14	Black	Neillsville[e]	756	1915–1942	0.60	0.783
15	Rib	Rib Falls[e]	309	1926–1942	0.71	0.982
16	Eau Claire	Kelly[e]	326	1915–1942	0.38	0.819
17	Embarras	Embarras[e]	395	1920–1942	0.35	0.764
18	Wolf	Keashena Falls[e]	812	1914–1942	0.20	0.958
19	Oconto	Gillett[e]	678	1915–1942	0.26	0.891
20	Peshtigo	High Falls[e]	571	1913–1942	0.36	0.871
21	Pike	Amberg[e]	250	1915–1942	0.25	0.932
22	Menominee	Grand Rapids[f]	3,790	1911–1929	0.29	0.897
23	Black	Galesville[e]	2,120	1933–1942	0.44	0.839
24	La Crosse	West Salem[e]	412	1915–1942	0.18	0.724
25	Little La Crosse	Leon[e]	77	1935–1944	0.15	....
26	Yellow	Sprague[e]	420	1927–1940	0.65	0.540
27	Waupaca	Waupaca[e]	305	1917–1942	0.15	0.830
28	Little Wolf	Royalton[e]	485	1915–1942	0.29	0.882
29	Wolf	New London[e]	2,240	1914–1942	0.28	0.820
30	Fox	Berlin[e]	1,430	1900–1942	0.24	0.785

TABLE 1.—(*Continued*)

Station No.	River	Station	Drainage area (sq miles)	Period of record	Flow variability index	Mean flow (cu ft per sec per sq mile)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
		(*b*) STREAMS IN WISCONSIN.—(*Continued*)				
31	Kickapoo	Gays Mills[e]	616	1915–1933	0.21	0.696
32	Wisconsin	Muscoda[e]	10,300	1915–1942	0.26	0.856
33	Crawfish	Milford[e]	732	1932–1942	0.57	0.399
34	Rock	Watertown[e]	971	1932–1942	0.68	0.401
35	Cedar Creek	Cedarburg[e]	113	1931–1942	0.70	0.485
36	Milwaukee	Milwaukee[e]	661	1915–1942	0.48	0.614
37	Grant	Burton[e]	257	1935–1944	0.25	....
38	Sugar	Brodhead[e]	529	1915–1942	0.24	0.655
39	Rock	Afton[e]	3,290	1914–1928	0.37	0.618
		(*c*) STREAMS IN ILLINOIS				
1	Pecatonica	Freeport[b]	1,340	1914–1928	0.26	0.766
2	Fox	Algonquin[g]	1,364	1915–1933	0.39	0.614
3	Rock	Lyndon[g]	9,090	1914–1928	0.30	0.694
4	Des Plaines	Lement[g]	687	1915–1933	0.72	0.657
5	Vermilion	Streator[g]	1,080	1915–1930	0.96	0.611
6	Kankakee	Custer Park[g]	4,870	1915–1923	0.42	0.640
7	Kankakee	Momence[g]	2,340	1915–1933	0.32	0.762
8	Spoon	Seville[g]	1,600	1915–1933	0.59	0.620
9	La Moine	Ripley[g]	1,310	1922–1933	0.65	0.604
10	Sangamon	Riverton[g]	2,560	1908–1933	0.77	0.696
11	Sangamon	Monticello[g]	550	1909–1933	0.76	0.763
12	Vermilion	Danville[g]	1,280	1919–1933	0.73	0.566
13	Macoupin Creek	Kane[g]	865	1921–1933	0.85	0.721
14	South Fork	Kincaid[g]	510	1917–1933	0.89	0.824
15	Kaskaskia	Vandalia[g]	1,980	1908–1933	0.70	0.753
16	Embarras	St. Marie[g]	1,540	1910–1933	0.69	0.823
17	Little Wabash	Wilcox[g]	1,130	1915–1933	0.87	0.712
18	Skillet Fork	Wayne City[g]	475	1909–1933	1.09	0.940
19	Big Muddy	Plumfield[g]	753	1915–1933	1.01	1.021
20	Cache	Forman[g]	240	1923–1933	1.02	1.400
		(*d*) STREAMS IN OHIO				
1	Maumee	Waterville[h]	6,314	1921–1940	0.79	0.671
2	Sandusky	Fremont[h]	1,248	1923–1940	0.63	0.702
3	Rocky	Berea[h]	269	1923–1935	0.81	0.948
4	Grand	Madison[h]	587	1922–1940	0.89	1.053
5	Ashtabula	Ashtabula[h]	118	1924–1940	1.17	1.144
6	Conneaut Creek	Amboy[h]	178	1922–1935	0.79	1.225
7	Maumee	Antwerp[h]	2,049	1921–1940	0.56	0.744
8	Auglaize	Defiance[h]	2,329	1915–1935	0.73	0.696
9	Auglaize	Fort Jennings[h]	333	1921–1935	0.56	0.883
10	Blanchard	Findlay[h]	343	1923–1935	0.79	0.656
11	Ottawa	Allentown[h]	168	1923–1935	0.62	0.684
12	Sandusky	Upper Sandusky[h]	299	1921–1940	0.75	0.833
13	Jerome Fork	Jeromesville[h]	120	1925–1939	0.63	0.883
14	Cuyahoga	Old Portage[h]	405	1921–1940	0.43	1.071
15	West Branch, Mahoning	Newton Falls[h]	98	1926–1939	0.55	0.939
16	Mosquito	Niles[h]	139	1929–1939	1.11	0.748
17	Mahoning	Youngstown	899	1903–1939	0.53	0.914
18	Scioto	Dublin[h]	988	1921–1939	0.69	0.784
19	Mohican	Greer[h]	942	1921–1939	0.46	0.937
20	Kokosing	Millwood[h]	472	1921–1939	0.49	1.023

TABLE 1.—(*Continued*)

Station No.	River	Station	Drainage area (sq miles)	Period of record	Flow variability index	Mean flow (cu ft per sec per sq mile)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
		(*d*) STREAMS IN OHIO.—(*Continued*)				
21	Walhonding	Nellie[h]	1,499	1910–1939	0.46	0.969
22	Tuscarawas	Newcomerstown[h]	2,439	1921–1939	0.47	1.007
23	Tuscarawas	Dover[h]	1,398	1923–1939	0.43	1.000
24	Sandy Creek	Sandyville[h]	481	1923–1939	0.41	1.065
25	Little Beaver Creek	East Liverpool[h]	505	1915–1939	0.57	1.020
26	Stillwater	Englewood[h]	646	1925–1939	0.56	0.904
27	Miami	Taylorsville[h]	1,155	1914–1939	0.54	0.846
28	Miami	Dayton[h]	2,513	1905–1939	0.44	0.921
29	Mad	Dayton[h]	632	1914–1939	0.30	1.050
30	Mad	Springfield[h]	485	1904–1939	0.29	1.053
31	Buck Creek	Springfield[h]	137	1914–1939	0.30	0.964
32	Scioto	Columbus[h]	1,624	1921–1939	0.62	0.843
33	Alum Creek	Columbus[h]	190	1923–1940	0.65	0.868
34	Big Walnut Creek	Rees[h]	544	1921–1939	0.64	0.917
35	Licking	Toboso[h]	672	1921–1939	0.50	0.992
36	Muskingum	Dresdon[h]	5,982	1921–1939	0.48	1.021
37	Muskingum	McConnelsville[h]	7,411	1921–1939	0.48	0.791
38	Twin Creek	Germantown[h]	275	1914–1939	0.61	1.045
39	Miami	Hamilton[h]	3,639	1910–1939	0.46	0.966
40	Little Miami	Milford[h]	1,195	1925–1939	0.58	1.098
41	Darby Creek	Darbyville[h]	533	1921–1939	0.65	0.818
42	Deer Creek	Williamsport[h]	331	1926–1939	0.67	0.867
43	Scioto	Chillicothe[h]	3,847	1913–1939	0.55	0.889
44	Hocking	Athens[h]	944	1915–1939	0.55	1.088
45	Whiteoak Creek	Georgetown[h]	221	1923–1935	1.00	1.150
		(*e*) STREAMS IN MAINE				
1	St. John	Fort Kent[i]	5,690	1926–1941	0.46	1.73
2	Fish	Fort Kent[i]	867	1929–1941	0.43	1.60
3	Allagash	Allagash[i]	1,250	1931–1941	0.47	1.62
4	Mattawamkeag	Mattawamkeag[i]	1,400	1913–1941	0.48	1.75
5	Piscataquis	Dover-Foxcroft[i]	286	1912–1941	0.55	2.16
6	Kennebec	Bingham[i]	2,660	1927–1941	0.47	....
7	Carrabassett	North Anson[i]	354	1925–1941	0.48	1.90
8	Sandy	Mercer[i]	514	1929–1941	0.48	1.72
9	Swift	Roxbury[i]	95	1929–1941	0.52	1.94
10	Passadumkeag	Lowell[i]	299	1915–1941	0.36	1.64
11	West Branch, Union	Amherst[i]	148	1929–1941	0.58	1.69
12	Machias	Whitneyville[i]	465	1929–1941	0.39	1.37
13	East Machias	East Machias[i]	251	1927–1941	0.46	1.98
		(*f*) STREAMS IN NORTH CAROLINA				
1	Roanoke	Old Gaston[j]	8,280	1911–1932	0.34	0.970
2	Cape Fear	Lillington[k]	3,530	1925–1935	0.59	0.876
3	French Broad	Asheville[k]	949	1925–1935	0.26	1.910
4	Broad	Boiling Springs[k]	815	1926–1935	0.25	1.727
5	Fishing Creek	Enfield[k]	462	1925–1935	0.47	0.992
6	Hiwassee	Murphy[k]	410	1925–1935	0.33	2.056
7	Deep	Ramseur[k]	343	1925–1935	0.51	0.991
8	Nottely	Ranger[k]	272	1925–1935	0.29	1.812
9	Second Broad	Cliffside[k]	230	1926–1935	0.27	1.264
10	Flat	Bahama[k]	150	1926–1935	0.62	0.893
11	Cullasaja	Cullasaja[k]	87	1925–1935	0.37	2.460

TABLE 1.—(*Continued*)

Station No.	River	Station	Drainage area (sq miles)	Period of record	Flow variability index	Mean flow (cu ft per sec per sq mile)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
		(*g*) STREAMS IN VERMONT				
1	Missisquoi	Richford[i]	479	1917–1942	0.48	....
2	Missisquoi	North Troy[i]	131	1932–1942	0.47	2.00
3	Lamoille	Johnson[i]	310	1929–1942	0.38	1.64
4	Lamoille	Cady's Falls[m]	250	1913–1923	0.38	1.73
5	Moose	St. Johnsbury[i]	126	1929–1941	0.49	....
6	Winooski	Essex Junction[n]	1,018	1909–1929	0.43	1.66
7	Winooski	Montpelier[n]	338	1909–1929	0.43	1.43
8	Otter Creek	Middlebury[i]	628	1911–1944	0.53	....
9	White	West Hartford[i]	690	1916–1942	0.45	1.67
10	Black	North Springfield[i]	158	1931–1942	0.44	....
		(*h*) STREAMS IN NEW HAMPSHIRE				
1	Connecticut	Orford[o]	3,100	1900–1915	0.40	1.712
2	East Branch of Pemigewasset	Lincoln[i]	104	1930–1942	0.42	....
3	Saco	Conway[i]	386	1929–1941	0.42	2.28
4	Pemigewasset	Plymouth[i]	622	1904–1942	0.43	....
5	Smith	Bristol[i]	85.8	1919–1942	0.48	....
6	Blackwater	Contoocook[i]	134	1919–1935	0.43	1.67
7	Contoocook	Penacook[i]	766	1930–1941	0.41	1.62
8	Ashuelot	Gilsum[i]	71.1	1923–1942	0.50	....
9	Ashuelot	Hinsdale[i]	420	1908–1942	0.45	....
10	Souhegan	Merrimack[p]	171	1910–1942	0.50	....
		(*i*) STREAMS IN MASSACHUSETTS				
1	Farmington	New Boston[i]	92	1914–1941	0.36	....
2	Middle Branch, Westfield	Goss Height[i]	52.6	1911–1941	0.58	....
3	Westfield	Knightville[p]	162	1911–1920	0.52	....
4	Manhan	Holyoke	13	1897–1910	0.51	2.03
5	Connecticut	Holyoke[p]	8,390	1880–1899	0.36	1.454
6	Connecticut	Sunderland[p]	8,000	1911–1920	0.39	....
7	Sip Pond Brook	Winchenden[i]	18.2	1917–1939	0.47	....
8	Merrimack	Lawrence[p]	4,552	1900–1920	0.37	1.430
9	Charles	Waltham[q]	227	1932–1942	0.54	....
10	Ipswich	Ipswich[i]	124	1931–1942	0.61	....
11	Moss Brook	Wendell Depot[i]	12.3	1917–1942	0.52	....
		(*j*) STREAMS IN PENNSYLVANIA				
1	Susquehanna	Harrisburg[r]	24,100	1890–1936	0.44	1.423
2	Muddy Creek	Castle Finn	133	1928–1938	0.29	1.162
3	West Conewago Creek	Manchester	510	1931–1941	0.58	1.120
4	Conodoguinet	Hogestown	470	1931–1941	0.40	1.086
5	Sherman Creek	Shermans Dale	200	1931–1941	0.50	1.265
6	Conestoga Creek	Lancaster	322	1926–1941	0.41	1.097
7	North Bald Eagle Creek	Beach Creek Station	559	1931–1941	0.40	1.172
8	Clearfield Creek	Dimeling	370	1932–1941	0.58	1.396
		(*k*) STREAMS IN KANSAS				
1	Kansas	Orden[s]	45,200	1918–1934	0.37	0.0443
2	Republican	Wakefield[s]	24,700	1919–1933	0.35	0.0397
3	Blue	Randolph[s]	9,100	1919–1934	0.39	0.132
4	Smoky Hill	Ellsworth[s]	7,580	1919–1934	0.50	0.0290
5	Solomon	Niles[s]	6,770	1920–1934	0.47	0.0435

TABLE 1.—(*Continued*)

Station No.	River	Station	Drainage area (sq miles)	Period of record	Flow variability index	Mean flow (cu ft per sec per sq mile)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
		(*k*) STREAMS IN KANSAS.—(*Continued*)				
6	Neosho	Parsons[s]	4,828	1922–1934	0.76	0.456
7	Saline	Tescott[s]	2,820	1920–1934	0.53	0.0497
8	Walnut	Winfield[s]	1,894	1922–1934	0.65	0.290
		(*l*) MISCELLANEOUS STREAMS				
1	Ohio	Paducah, Ky.	203,000	....	0.44	....
2	Colorado	Austin, Tex.	37,000	....	0.59	....
3	Tennessee	Florence, Ala.[t]	30,800	30 years	0.37	1.694
4	Coosa	Childersburg, Ala.[u]	8,390	1896–1912	0.40	1.680
5	Choccolocco Creek	Jenifer, Ala.[v]	275	16 years	0.35	....
6	Locust Creek	Trafford, Ala.[v]	622	13 years	0.72	....
7	Delaware	Port Jervis, N. Y.[w]	3,070	26 years	0.44	1.825
8	Hudson	Spier Falls, N. Y.[x]	2,777	1888–1910	0.31	....
9	Delaware	Fish Eddy, N. Y.	875	27 years	0.47	2.331
10	Delaware	Trenton, N. J.	6,800	17 years	0.35	1.690
11	Clinch	Coal Creek, Tenn.[t]	2,960	1904–1934	0.45	1.504
12	Hiwassee	Reliance, Tenn.[t]	1,220	30 years	0.31	2.190
13	Etowah	Allatoona Creek, Ga.[u]	1,100	1896–1929	0.34	2.053
14	Coosawattee	Carters, Ga.[u]	530	1896–1929	0.30	2.253
15	Roanoke	Roanoke, Va.[j]	388	1896–1931	0.39	1.030
16	Dan	South Boston, Va.[j]	2,730	1901–1931	0.30	1.050
17	Elkhorn	Waterloo, Nebr.[y]	6,390	10 years	0.28	0.108
18	Middle Loup	St. Paul, Nebr.[y]	7,320	10 years	0.19	....
19	Loup	Columbus, Nebr.[y]	14,200	10 years	0.17	0.164
20	St. Lawrence	Below Lake Ontario	286,688	1894–1925	0.045	....
21	St. Clair	Below Lake Michigan	213,941	1897–1925	0.025	....
22	St. Marys	Below Lake Superior	76,136	1897–1925	0.058	...
23	Muskegon	Croton Dam, Michigan	....	1910–1919	0.22	....

[a] "Surface Water Resources of Iowa," *Water Supply Bulletin No. 2*, Iowa Geological Survey, Iowa City, Iowa. [b] "Des Moines River, Iowa," *House Document No. 682*, 71st Cong., 3d Session, Washington, D. C., 1931. [c] "Iowa River, Iowa and Minnesota," *House Document No. 134*, 71st Cong., 2d Session, Washington, D. C., 1930. [d] "Summaries of Yearly and Flood Flow Relating to Iowa Streams," *Water Supply Bulletin No. 1*, Iowa Geological Survey, Iowa City, Iowa, 1942. [e] U. S. Geological Survey, Madison, Wis. [f] "Menominee River, Michigan and Wisconsin," *House Document No. 141*, 72d Cong., 1st Session, Washington, D. C., circa 1935. [g] "Stream Flow Data of Illinois," Dept. of Public Works, Springfield, Ill., 1937. [h] "Ohio Stream Drainage Areas and Flow Duration Tables," by C. H. Hall and C. V. Youngquist, *Bulletin No. III*, Ohio State Univ. Experiment Station, Columbus, Ohio, May, 1942. [i] Tenth Biennial Rept, Public Utilities Comm., State of Maine, 1941–1942. [j] "Roanoke River, Va. and N. C.," *House Document No. 65*, 74 Cong., 1st Session, Washington, D. C., 1935. [k] "Discharge Records of North Carolina Streams," *Bulletin No. 39*, Dept. of Conservation and Development, North Carolina, Raleigh, N. C., 1938. [l] U. S. Geological Survey, Boston, Mass. [m] "Lamoille River, Vt," *House Document No. 145*, 72d Cong., 1st Session, Washington, D. C. [n] "Winooski River, Vt.," *House Document No. 785*, 71st Cong., 3d Session, Washington, D. C., 1931. [o] "Rainfall and Run-Off Studies," by C. E. Grunsky, *Transactions*, ASCE, Vol. LXXXV, 1922, p. 66. [p] *Journal*, Boston Soc. of Civ. Engrs., Vol. 9, 1922. [q] "Computing Run-Off from Rainfall and Other Physical Data," by Adolph F. Meyer, *Transactions*, ASCE, Vol. LXXIX, 1915, p. 1056. [r] "The Drought of 1930 in Pennsylvania," by J. W. Mangan, Div. of Hydrography, Harrisburg, Pa., 1937. [s] "Notes on the Hydrology of Kansas," *Engineering Bulletin No. 20*, Univ. of Kansas, Lawrence, Kans., 1936. [t] Tennessee Valley Authority. [u] "Alabama-Coosa Branch of Mobile River System," *House Document No. 66*, 74th Cong., 1st Session, Washington, D. C., 1935. [v] U. S. Geological Survey, Montgomery, Ala. [w] "Report of Precipitation, Flood, and Reservoir Studies," Iowa Inst. of Hydr. Research, Iowa City, Iowa. [x] Sixth Annual Rept. of the State Water Supply Comm. of New York, Albany, N. Y. [y] "Water Resources of Nebraska," Nebraska State Planning Board (revised February, 1941).

to those in the adjacent rough part of Iowa. The lake regions in the northern part of Wisconsin also produced low values ranging from 0.14 to 0.38. The great moraines of the Wisconsin glacier are largely effective in producing uniform flows, which are indicated by the low values of 0.15 to 0.35. The flat regions, without lakes or moraines, gave rise to values of 0.53 to 0.71 as in similar regions in Iowa. Large variations in value occurred within small distances, much too small to be accounted for by variations in rainfall. The only satisfactory explanation seemed to be surface storage or geological conditions.

Only a few of the 20 stations analyzed in Illinois (Table 1(*c*)) were affected by surface storage. The terminal moraines of the glaciers, however, exercised a great influence on many streams. The areas where lakes or extensive moraines were not present, but were underlain by impervious strata, produced high values of the coefficient ranging from 0.69 to 1.09. In Illinois, geology also seemed to be the principal cause, although there is a possibility that rainfall variation may have had some influence in producing the high values in the southern part of the state.

The 45 stations analyzed in Ohio (Table 1(*d*)) showed results similar to those in Iowa, Wisconsin, and Illinois. The values obtained range from 0.29 to 1.17, the latter being the highest value found in the entire study. One of the interesting developments was the differences in the indexes of the tributaries of the Miami River. Three of the main tributaries gave values ranging from 0.54 to 0.61, and one, the Mad River, had a value of 0.30. The main stream below the tributaries had values of from 0.44 to 0.46, very close to what would be obtained from a weighted mean of the tributary flows.

The low values for Mad River resulted from an unusually large area of glacial moraine in its watershed, together with porous bedrock and the bed of a large preglacial river. Except for geology, its basin does not differ noticeably from those of the other tributaries.

In Maine (Table 1(*e*)) 13 stations were found where artificial regulation was negligible, giving values ranging from 0.36 to 0.58. Lakes and swamps exercised an important influence on many of these data. The equalizing effect of a given area of lakes in this state did not appear to be so great as in Wisconsin, probably because of the more impervious soil and bedrock conditions in Maine.

There was some variation in the results obtained for the 32 stations analyzed in Vermont, New Hampshire, and Massachusetts (Tables 1(*g*), 1(*h*), and 1(*i*), respectively), the values varying from 0.36 to 0.61. Sufficient data were not available to analyze the causes of these variations.

The 11 stations studied in North Carolina (Table 1(*f*)) ranged from 0.25 to 0.62, the high values being in the flatter section of the state and the low values, in the mountains. The low mountain values probably reflect the effect of rainfall. In these mountains rain occurs very frequently, which tends to produce uniform flows.

The flow of 8 streams in the Susquehanna Valley in Pennsylvania (Table 1(*j*)) were studied because stream flow variability was known to change widely within short distances because of geological conditions. The values varied from 0.29 to 0.58, under conditions that seemed to preclude any other cause than geology.

Analysis was made of 8 streams in Kansas (Table 1(*k*)), although the rainfall on these watersheds was usually much less than that on the other streams investigated in this study, and the variability values did not differ greatly, the values ranging from 0.35 to 0.76. Some of these streams in the Great Plains extended out to, or nearly to, the Rocky Mountains. Sufficient data were not available on the geology of their basins to determine its effect on the flow variability.

Data were available from 23 miscellaneous rivers, extending from Georgia and Texas to Nebraska (Table 1(*l*)). To show the lower limit of the variability index, the flows of the St. Lawrence, St. Clair, and St. Marys rivers in the Great Lakes watershed were analyzed, giving values of 0.045, 0.025, and 0.058, respectively (based on monthly mean flows). The effect of extensive deposits of sand in the watershed in equalizing the water flow was shown by the values obtained on the Loup and Elkhorn rivers of Nebraska and the Muskegon River in Michigan, which gave values of from 0.17 to 0.28. These were among the owest values obtained.

Use of the Variability Index

One of the principal uses of the index described in this paper is that it enables the variability of streams to be compared readily by giving numerical values to this characteristic of their flow. Usually only general terms have been used for such comparisons in the past, and only very rough comparisons were possible. The agreement of the flow duration curve with a straight line on logarithmic probability paper offers a convenient method of comparing the form of the flow duration curves.

Another use of the index is that it has revealed the effect of the several factors on the variability of the flow of streams during the greater part of the time (but not in floods or droughts). Of the seven factors listed by Mr. Morgan,[9] this study has shown that geology (including soil conditions) and surface storage in lakes and swamps are the predominant factors affecting the variation of ordinary flows in most of the streams investigated. Although data were not available to indicate conclusively how large was the effect of rainfall variability, the evidence was strong that it was usually less important than geology or surface storage.

Probably the greatest use of this index will be in developing synthetic duration curves for the analysis of hydraulic engineering problems, where the available data are meager. It can thus be used in studies of water power, water storage, and sediment production, where duration curves have been found to be useful. The method consists of:

(1) Selecting a variability coefficient depending on the characteristics of the watershed;

(2) Determining the mean annual runoff of the stream; and

(3) Drawing the line on logarithmic probability paper which is represented by the variability index and the mean annual runoff.

If stream flow records are available for watersheds in the vicinity, having similar characteristics to the one under study, the value obtained for this stream can be used. If the difference between the watersheds is small, the value desired may be obtained by applying a small correction to the known value.

If stream flow records of short duration are available, they may be used and compared with the variability values obtained from records of streams in the vicinity, both for the same and longer periods, if possible. For cases where no flow records are available, the following method has been developed from a study of all the variability index values included in Table 1 and the geological and surface storage conditions of the watersheds. The values given in Table 2 have been prepared for use in selecting a variability coefficient, assuming that the index for an average river is 0.60 and that the presence of other conditions in the watershed, indicated in Table 2, give rise to the corrections listed, which

TABLE 2.—CHARACTERISTIC VALUES OF VARIABILITY COEFFICIENT

Terrain	Description	Subtract from 0.60	Add to 0.60
Bedrock	Impervious	....	0.1 to 0.2
	Average	0.1 to 0	0 to 0.1
	Pervious	0.1 to 0.2	....
Soil, deep covering	Impervious	....	0.1 to 0.2
	Average	0.1 to 0	0 to 0.1
	Pervious	0.1 to 0.2	....
	Very pervious	0.2 to 0.3	....
	Sand hills	0.3 to 0.4	....
Relief	Flat or gently rolling	0	0
	High in impervious material	0.05 to 0.1	....
	High in average material	0.1 to 0.2	....
	High in pervious material	0.2 to 0.3	....
Moraines	Impervious	0	0
	Average	0.05 to 0.1	....
	Pervious	0.1 to 0.2	....
	Pervious, with many kettle holes	0.2 to 0.4	....
Lakes and swamps	Small areas	0.05 to 0.1	....
	Moderate areas	0.1 to 1.5	....
	Extensive areas	1.5 to 2.5	....

are either added to, or subtracted from, the value of 0.60. The corrections for soil and bedrock are intended to be applied only to flat or gently rolling land and not to rugged areas. The values for moraines and deep soils should be weighted according to the proportion of the watersheds covered by them. Where the soil cover is deep, the bedrock corrections should be reduced, as the influence of bedrock is less. In a terrain covered with moraines, and also containing lakes or swamps, the highest values of the corrections given should not be used.

The streams having large subsurface storage frequently are somewhat skewed with a curve that is convex to the origin of coordinates. This gives higher flows for the low and high values than the straight line would indicate and smaller flows for the medium. As previously stated, the very high discharge values from the straight line are too high and the very low values are too low since the straight line extends to both zero and infinity.

Until considerable comparison has been made, it will not be possible to judge how far these values can be applied in regions other than the northeastern United States. In a region where data are meager, the variability values for such data as are available should be obtained, and these relations tested out on it.

In applying these values, it will be necessary to use judgment; otherwise negative values may be obtained. For example, the bedrock and relief corrections should not both be used as the bedrock corrections are included in those for relief. The correction for moraines should apply to the area covered by moraines and the soil corrections should not be applied to this area also. Where bedrock lies so deep that it is below the water table, it should not be considered.

Large watersheds will tend to have lower values than small ones, because the runoffs from rainfall from the various parts of the basin will not synchronize, and because the channel storage will tend to equalize the flow.

The method of determining the mean annual runoff of streams will depend upon the amount of data available, the more data the more accurately the runoff can be estimated. If no rainfall or stream flow data are available, the runoff can be estimated roughly from that in other localities producing similar vegetation. This method, of course, is very inexact and should be used only as a last resort. If rainfall data only are available, the runoff can be estimated by estimating the loss and subtracting it from the rainfall. The loss can be estimated from that of watersheds having similar conditions; or, if temperature records are available, by the methods of Adolph F. Meyer,[19] M. ASCE, or J. W. Johnson,[18] M. ASCE. In the eastern half of the United States, the loss may be estimated from data supplied by the U.S.G.S.[20] If a short term of runoff record is available on the watershed under study, its runoff could be compared with that for the same years of a watershed in the vicinity that had a long record and also with the runoff of this latter watershed for the total period of record.

The shape of the duration curve can be obtained by drawing a straight-line duration curve on logarithmic probability paper with a slope such that the ratio of the discharge exceeded 15.87% of the time to the discharge exceeded 50% of the time was equal to the variability index selected. The ends of this line for a short distance on each end will have to be altered to match the best possible estimates of maximum and minimum flow, since a straight line on this paper goes from infinity to zero. A considerable error in these estimates will not have much effect, as they affect only a small part of the time. The position of the curve will have to be established by trial, making the area under the curve (when plotted on coordinates similar to Fig. 1) represent the quantity of water indicated by the mean annual runoff.

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[19] "Computing Run-Off from Rainfall and Other Physical Data," by Adolph F. Meyer, *Transactions*, ASCE, Vol. LXXIX, 1915, p. 1056.

[20] "Natural Water Loss in Selected Drainage Basins," by G. R. Williams and others, *Water-Supply Paper No. 846*, U.S.G.S., Washington, D. C., 1940.

Possible Future Studies

Further studies will probably show that this variability index is directly related to such factors as the magnitude of minimum flows, of flood flows of moderate frequency, and the storage necessary to maintain various uniform flows. There is good reason to believe that the greater the variability index, the greater will be the spread between the duration curves of individual years. If this be true, a more detailed estimate could be made of the frequency of periods of various magnitude of flow than it is possible to obtain from a simple duration curve.

From a study of the relation of the watershed characteristics and the departure of some of the curves from a straight line on logarithmic probability paper, it may be possible to determine the conditions giving rise to these departures and thus enable more accurate synthetic curves to be developed than would be possible by using the straight-line method.

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PAPERS

END RESTRAINTS ON TRUSS MEMBERS

By Harold E. Wessman,[1] M. ASCE, and Thomas C. Kavanagh,[2] Assoc. M. ASCE

Synopsis

One of the problems of current interest to structural engineers is the behavior of compression members that are parts of rigidly connected structural frames and are consequently subjected to varying end restraints. A method is presented herein for the calculation of these end restraints and the effective lengths of compression members under different load conditions. It is demonstrated that the end restraints decrease and that the effective lengths increase as loads are increased, until the frame buckles. It is shown that when a truss buckles all members tend to fail simultaneously.

The procedure is an extension of the moment distribution method introduced by E. E. Lundquist, Assoc. M. ASCE, for analyzing the stability of a structural frame. The relation between the series criterion and the stiffness criterion for testing stability is restated. General equations are then formulated for the stiffness of an axially loaded bar with the far end elastically restrained. The general equation for the instability of a bar with known elastic restraints is also derived in terms of the moment distribution method. The procedure is illustrated by the calculations for a specific truss.

Finally, an important fact which is not well understood is presented—namely, that for compression members in steel trusses with the usual range of slenderness ratios, the buckling load coincides practically with the yield point of the steel regardless of end restraints. Hence any elaborate analysis of stability and end restraints is unwarranted for a steel truss.

The behavior of a compression member or column as a part of a riveted or welded truss or rigid frame is a subject of current interest to a growing number

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of structural engineers. Current design practice is not very consistent in the treatment of restraints existing at the ends of a column by virtue of its more-or-less rigid connection with other members of varying stiffness. Some widely accepted column formulas contain factors for end conditions, whereas other equally accepted formulas do not. Most practicing engineers have little knowledge of these restraints, beyond the usual textbook statements that a pin-ended column corresponds to a length reduction factor of $k = 1$, whereas a fixed-ended column corresponds to $k = \frac{1}{2}$, in the Euler formula for the critical load P_{cr} in the elastic range:

$$P_{cr} = \frac{\pi^2 E I}{(k L)^2} \quad \ldots\ldots (1)$$

in which k is a factor depending on the degree of restraint at the ends; E is the modulus of elasticity; I is the moment of inertia of the column cross section; and L is the length of the column.

In the elastic region, consequently, a fixed column has four times as much capacity with respect to buckling as does the same pin-ended column. This condition is not true, however, in the inelastic range. The compression member usually encountered in civil engineering practice has such a low slenderness ratio that it acts in the inelastic range as load is increased and approaches the critical value. Eq. 1 then becomes:

$$P_{cr} = \frac{\pi^2 E' I}{(k L)^2} \quad \ldots\ldots (2)$$

in which E', the effective modulus, replaces E, the elastic modulus. The tangent modulus is now generally accepted by aeronautical engineers as the effective modulus[3] for columns. It has also been accepted by the Column Research Council of Engineering Foundation. For structural steel, E' gradually decreases from 30×10^6 lb per sq in. to zero at the yield point, a value that may be termed a "ceiling" for the critical average stress. As the slenderness ratio is decreased, the Euler load curves based on the tangent modulus gradually converge toward this ceiling. It will be found then that the buckling load for a fixed-end column may be only slightly greater than that for the same column with pinned ends. The ratio is no longer 4; it decreases rapidly to values less than 2 and very close to 1.

In the actual column as it exists in a framework, the ends are neither pinned nor fixed; in other words, the end restraints are neither zero nor infinity, but have some intermediate values best characterized by rotational springs having spring constants s (see Fig. 1). In this type of column, the critical buckling load will have a value which is between that for the pinned column (the lower limit) and that for the fixed column (the upper limit). The reader should bear in mind that these two limits are not far apart for columns with low slenderness ratios. Moreover, for structural steel, they are very close to the yield point of the steel.

It is not generally recognized that, as loads are increased on a framework, the end restraints acting on a compression member in the framework do not

[3] "Inelastic Column Theory," by F. R. Shanley, *Journal of the Aeronautical Sciences*, May, 1947, p. 261.

remain constant. They may vary between almost absolute fixation and almost perfect freedom, depending on the changes in stiffness of the abutting members. In fact, the abutting members, instead of creating an end moment on the compression member as it starts to buckle, may have moments presented to them by the buckling member. This change of restraint, as a framework is loaded, is the reason why it is difficult to test an isolated column in a testing machine and to duplicate the variable end conditions obtaining during the test of a framework.

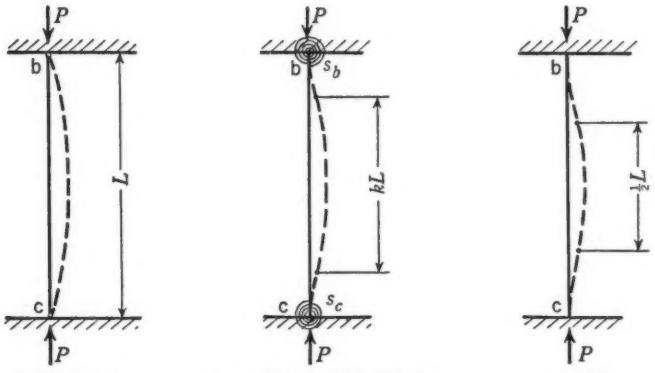

FIG. 1

The theoretical procedures for determining critical buckling loads for trusses are well established. One excellent method which is widely known among American aeronautical engineers is that developed by Mr. Lundquist.[4] This method is an adaptation of the moment distribution method developed by Hardy Cross,[5] Hon. M. ASCE. In the procedure of convergence to an accurate answer, stiffnesses of members are usually based on the condition that the far end is temporarily fixed at any one stage in the process. The actual end restraints acting on any member for any particular load condition are not usually evaluated in the Lundquist method. It is possible, however, to extend the procedure to determine the actual magnitude of the restraints and then to evaluate the effective length of a member corresponding to a particular load condition. There is some question as to the significance of "effective length" in column design formulas, inasmuch as the effective length changes as loads are increased. That question, however, will not be discussed in detail in this paper. The Lundquist method will now be restated briefly and then extended to the calculation of actual end restraints and effective lengths.

In stability calculations employing the moment distribution procedure, the stiffness and carry-over factor depend not only upon $E' I/L$ but upon the magnitude and sense of the axial load in the member. Evidently, an axial

[4] "Stability of Structural Members Under Axial Load," by E. E. Lundquist, *Technical Note No. 617*, National Advisory Committee for Aeronautics, Washington, D. C., October, 1937.

[5] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, ASCE, Vol. 96, 1932, p. 1.

tensile load will tend to increase the stiffness of a member, whereas an axial compressive load will reduce its stiffness. The expressions for stiffness and carry-over factor for axially loaded bars are well known, and are reproduced in the Appendix.

The stiffness of a bar on unyielding supports at each end is defined in the Lundquist method as the moment at one end necessary to produce a rotation of $\frac{1}{4}$ radian at that end. The stiffness obviously depends on the restraint at the far end, and three cases are considered: S is the stiffness, far end fixed; S' is the stiffness, far end elastically restrained; and S'' is the stiffness, far end pinned. The restraint created at the end of a member by an abutting member or members will be designated hereafter by the spring constant s, defined as the moment necessary to produce a rotation of $\frac{1}{4}$ radian in the spring. Accordingly, s corresponds to $\Sigma S'$, the combined stiffness of all adjacent members with their far ends elastically restrained.

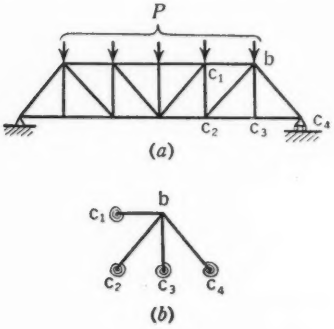

FIG. 2

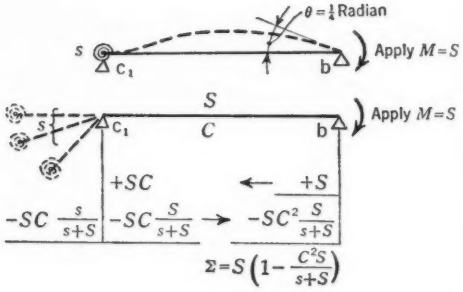

FIG. 3

The carry-over factor is employed in its usual sense with relation to a member on unyielding supports, as the ratio of the moment developed at the far end to the moment applied at the near end. Its value (C, C', or C'') also depends on the restraints at the far end.

If the values[6] of the stiffness and carry-over factors for the condition of "far end fixed" have been obtained, it is possible, by direct moment distribution, to obtain the factors for the other end conditions as well. Thus, for the stiffness S' of the bar bc_1 of the truss in Fig. 2, isolate the bar as shown in Fig. 3, where the bars at the left of c_1 have been denoted by an equivalent spring of total stiffness s. From the simple moment distribution shown, and the definitions of S' and C',

$$S' = S - \frac{C^2 S^2}{s + S} = S\left(1 - \frac{C^2 S}{s + S}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

and

$$C' = \frac{C}{1 + \frac{S}{s}(1 - C^2)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

[6] "Extended Tables of Stiffness and Carry-Over Factor for Structural Members Under Axial Load," by E. E. Lundquist and W. D. Kroll, *Wartime Report L-255* (ARR 4B 24), National Advisory Committee for Aeronautics, Washington, D. C., February, 1944.

If, now, a wire model of a rigid-jointed truss (Fig. 2) is visualized as being in stable equilibrium under the action of some external force system P, and if to any joint b of this truss an arbitrary small rotation θ_b is applied and held by hand, the structure will assume a new configuration in which each joint of the truss will be rotated through some angle that will be a multiple of θ_b, and the magnitude of which will depend on the stiffness of every member of the truss. If, as was assumed, the truss is in stable equilibrium, all such rotations will be finite, and on release of the joint, the truss will snap back to its original position.

As the load system P is increased, resulting in larger loads in the members of the truss, the stiffness of the individual members will be decreased or increased depending on whether they are in compression or in tension, respectively, and the resultant total stiffness of each joint will vary with increasing load. For a joint dominated by compression members, the total stiffness will decrease with increasing loads P, and the application of the small arbitrary rotation θ_b will cause increasingly larger rotations (or, in the model, the springiness or snapback of the joint on release will diminish continuously). Obviously, the criterion for stability is that the rotations $\left(\theta = \frac{M}{\Sigma S'}\right)$ at all joints be finite; this leads to the condition for neutral stability which Mr. Lundquist termed as the "stiffness criterion":

$$\Sigma S'_{bc} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

or

$$\Sigma S'_{bc} = \Sigma \left[S_{bc} \left(1 - \frac{C^2_{bc} S_{bc}}{s_c + S_{bc}} \right) \right] = 0 \ldots\ldots\ldots\ldots\ldots (6)$$

The stiffness criterion is usually difficult to apply, because the values of s are unknown, as a result of the unknown end restraints. It is applied indirectly, however, in the following manner: Eq. 3 states that the stiffness S' of a bar bc is equal to the stiffness S minus a correction, $\frac{C^2 S^2}{s + S}$, the latter being physically the moment carried back to (or the unbalanced moment at) joint b after the far end c has been balanced. It must be kept in mind that an initial moment of S was applied to bar bc at joint b, after which joint b was released and reclamped. If a number of members bc meet at joint b (Fig. 2(*b*)), all the S'_{bc}-values can be obtained by applying a moment ΣS_{bc} to the entire joint b, releasing and reclamping the joint. Then the process of moment distribution is applied to all the remaining joints only, using a sufficient number of cycles to insure accuracy. Finally, the moment values that have been carried back to joint *b* for each individual member meeting at b are summed. This summation for each member is the term $\frac{C^2 S^2}{s + S}$.

In practice, it is simpler to apply a unit moment to joint b rather than a moment of ΣS_{bc}. Accordingly, the initial moment at the end b of a member bc_1 is $\frac{S_{bc1}}{\Sigma S_{bc}}$ (see Fig. 3). The total moment carried back, designated as r_1 to

conform to the Lundquist notation, is then

$$r_1 = \frac{C^2_{bc1}\, S^2_{bc1}}{s_{c1} + S_{bc1}} \frac{1}{\Sigma S_{bc}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

The moment carried back to end b of member bc_2 is r_2. The total for all the members meeting at b is termed r and it is equal to $r_1 + r_2 + r_3 + \cdots$. It is the total unbalanced moment at joint b after all the other joints have been successively released, balanced, and clamped for several cycles of moment distribution. From Eq. 7,

$$r_1 \Sigma S_{bc} = \frac{C^2_{bc1}\, S^2_{bc1}}{s_{c1} + S_{bc1}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Comparing Eq. 8 with Eq. 3, the latter can be restated as follows for the members meeting at joint b:

$$\left.\begin{aligned} S'_{bc1} &= S_{bc1} - r_1 \Sigma S_{bc} \\ S'_{bc2} &= S_{bc2} - r_2 \Sigma S_{bc} \\ S'_{bc3} &= S_{bc3} - r_3 \Sigma S_{bc} \\ &\ldots\ldots\ldots \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Thus, an alternative to Eq. 3 is available for obtaining the stiffness at one end of a member with the far end elastically restrained. In the process of moment distribution, r_1, r_2, etc., are obtained directly for the members at joint b rather than the spring restraints s_{c1}, s_{c2}, etc., in Eq. 3. Once the corrected values of stiffness are obtained for the ends of the members adjacent to joint b, the stiffnesses at both ends of the remaining truss members can be obtained by repeated application of Eq. 3, solving either for an unknown S' in terms of the s at the far end, or for an unknown s in terms of a known S'.

The total stiffness at joint b with the far ends restrained elastically is the summation of Eqs. 9, or

$$\Sigma S'_{bc} = (1 - r)\, \Sigma S_{bc} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

At the critical or buckling load, Eq. 10 is set equal to zero, according to the stiffness criterion, and this immediately gives the very important condition for neutral stability, described by Mr. Lundquist as the "series criterion":

$$r = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The series criterion derives its name from the fact that the unbalanced moment at the joint is initially 1; after one release and reclamping, it is r; after the second release and reclamping it is r^2; etc. The final moment is then $1 + r + r^2 + \cdots$, which is a geometric series whose sum is $\frac{1}{1 - r}$; this moment is finite (and the structure is stable) as long as $r < 1$.

In determining the critical buckling load for a truss, the series criterion is more readily applied than is the stiffness criterion. With a given load condition acting on the truss, a unit moment is applied to any joint of the truss.

The joint is released, then clamped, and the total moment r, carried back to the joint through its abutting members, is calculated after all the remaining joints of the truss have been balanced by the application of several cycles of the moment distribution process. If $r < 1$, the truss is stable under the given load condition. The loading is increased and the procedure repeated, resulting in a new value for r. The determination of the buckling load for the truss—that is, the load at which $r = 1$—will require the analysis of a number of loading conditions, the number depending on the degree of accuracy sought.

The exact stiffness and the spring restraint for each end of any member for any one load condition, however, are quickly determined from Eqs. 9 and 3 after the values r_1, r_2, r_3, etc., have been found in determining r at a particular joint.

It is perhaps symbolic of the universal adaptability of the moment distribution method that it also affords a procedure for determining the value of the length reduction factor k from the two end restraints on a column. Considering the elastically restrained column bc (Fig. 1), and applying the stiffness criterion to joint b,

$$\Sigma S'_b = s_b + S'_{bc} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (12a)$$

from which, using Eq. 3,

$$s_b + S_{bc} - \frac{C^2_{bc}\, S^2_{bc}}{s_c + S_{bc}} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (12b)$$

Eq. 12*b* simplifies to

$$(s_b + s_c) + \frac{s_b\, s_c}{S_{bc}} + S_{bc}\,(1 - C^2_{bc}) = 0 \quad \ldots\ldots\ldots\ldots (12c)$$

The last term on the left-hand side of Eq. 12*c* is S''_{bc}, however, which is the stiffness with the far end pinned. The final result is

$$(s_b + s_c) + \frac{s_b\, s_c}{S_{bc}} + S''_{bc} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (13a)$$

For ease in application, Eq. 13*a* is rewritten in terms of the nondimensional restraint coefficients:

$$\left(\frac{s_b}{E'\,I/L} + \frac{s_c}{E'\,I/L}\right) + \frac{\dfrac{s_b}{E'\,I/L} \times \dfrac{s_c}{E'\,I/L}}{\dfrac{S_{bc}}{E'\,I/L}} + \frac{S''_{bc}}{E'\,I/L} = 0 \quad \ldots\ldots (13b)$$

Eq. 13*b* can be solved easily by trial from the tables[6] previously mentioned, since these tables give corresponding values of $\dfrac{S}{E'\,I/L}$ and $\dfrac{S''}{E'\,I/L}$ for values of L/j, in which $j = \sqrt{\dfrac{E'\,I}{P}}$. When the proper value of L/j has been found which satisfies Eq. 13*b*, the value of k is determined from Eq. 2 as

$$k = \frac{\pi}{L/j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

A trial solution is necessary because Eq. 13*a* is fundamentally a transcendental equation, since the functions S and S'' are trigonometric.[7] A convenient chart which gives a direct solution has been published by N. J. Hoff,[8] and is reproduced in Fig. 4, with the same notation as in this paper. Eq. 13*a* is merely one form of the fundamental equation of an elastically restrained bar, which was probably first stated by H. Zimmermann.[9] W. R. Osgood,[10] M. ASCE, was perhaps the first to note the equivalence between Mr. Zimmermann's basic equation and the moment distribution concepts employing stiffness and carry-over factors.

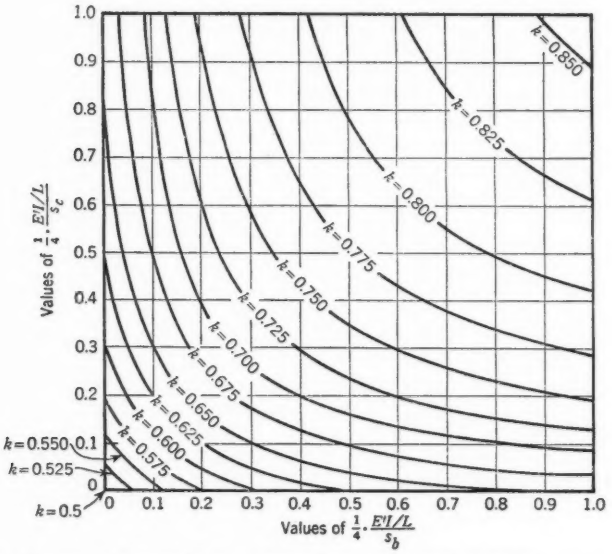

FIG. 4.—EFFECTIVE LENGTH FACTORS k FOR COLUMNS WITH END RESTRAINTS s_b AND s_c

The stability criteria first stated by Mr. Lundquist were given rigorous proof by Mr. Hoff [11,12] employing energy methods. By this latter procedure, Mr. Hoff was able to demonstrate that, for a truss to be stable, not only must the moment distribution converge to finite values, but the final values must be unique (that is independent of the order of balancing).

The procedure of finding end restraints and effective lengths will now be illustrated for the truss shown in Fig. 5(*a*). This truss is a "textbook" illus-

[7] "Principal Effects of Axial Load on Moment Distribution Analysis of Rigid Structures," by B. W. James, *Technical Note No. 534*, National Advisory Committee for Aeronautics, Washington, D. C., July, 1935.

[8] "Elastically Encastred Struts," by N. J. Hoff, *Journal*, Royal Aeronautical Soc., September, 1936, p. 663.

[9] "Knickfestigkeit der Stabverbindungen," by H. Zimmermann, W. Ernst & Sohn, Berlin, 1925.

[10] Discussion by W. R. Osgood of "Principles of Moment Distribution Applied to Stability of Structural Members," by E. E. Lundquist, *Proceedings*, 5th International Cong. for Applied Mechanics, Cambridge, Mass., 1938, pp. 149–150.

[11] "Stable and Unstable Equilibrium of Plane Frameworks," by N. J. Hoff, *Journal of the Aeronautical Sciences*, January, 1941, p. 115.

[12] "Stress Analysis of Aircraft Frameworks," by N. J. Hoff, *Journal*, Royal Aeronautical Soc., July, 1941, p. 241.

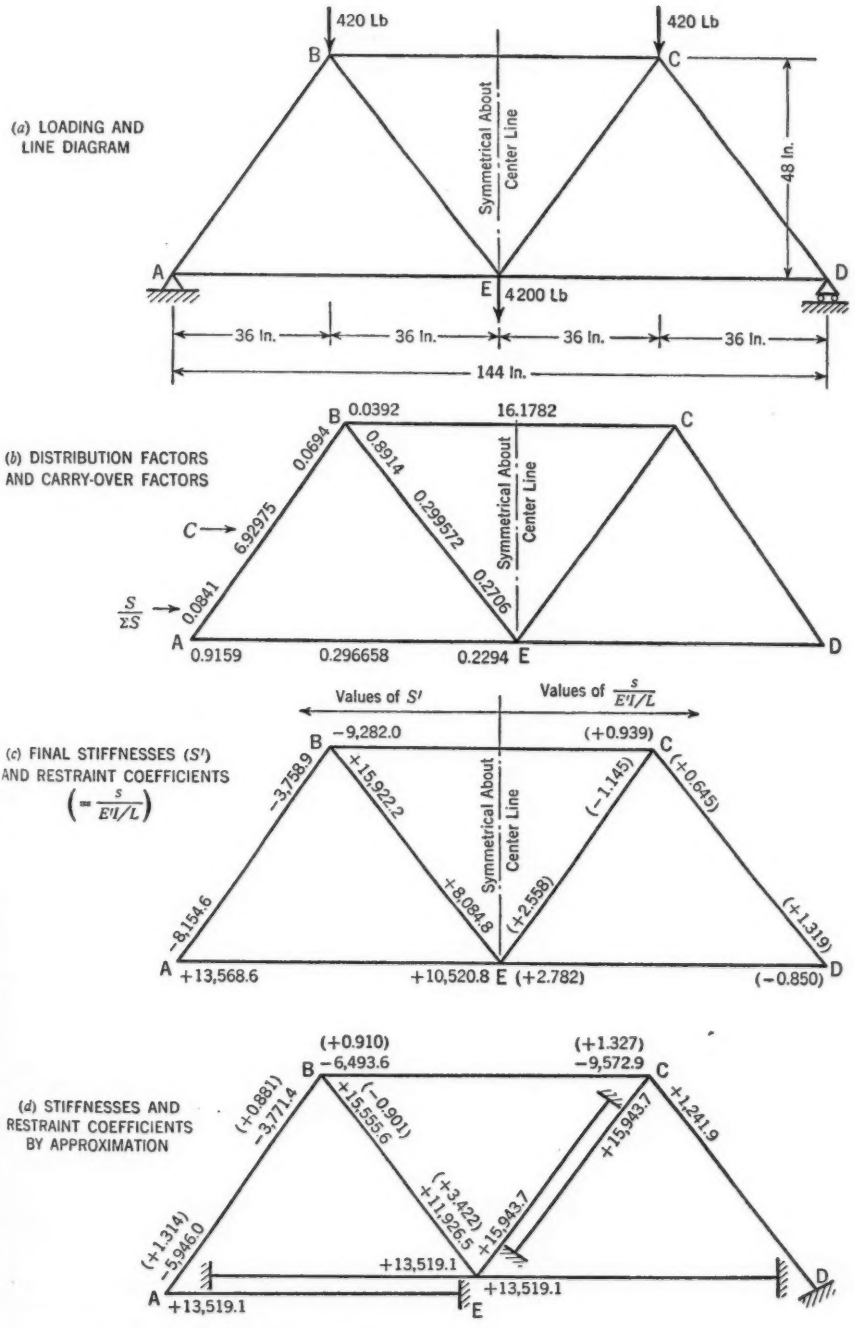

FIG. 5.—END RESTRAINTS AND EFFECTIVE LENGTHS; AN ILLUSTRATIVE EXAMPLE

tration,[13] adopted from the aircraft field. It has been designed to fail at the loads shown, but actually it has some reserve of strength above these design ultimate loads. The following questions might arise:

1. At what multiple of the load will the truss become unstable? (To answer this question would involve a series of trials. Assume, as a first trial value, $\frac{425}{420} = 1.0119$ times the load shown—that is, the loads at joints B and C are changed to 425 lb; and the load at joint E, to 4,250 lb.)

2. For any given multiple of the load (say, 1.0119) what are the end restraints on the bars?

3. Having the restraints, what are the corresponding effective lengths?

All three of these questions are answered by the procedure outlined previously.

TABLE 1.—STIFFNESS AND CARRY-OVER FACTORS FOR
(ALL UNITS IN TERMS OF

Member	P	P/A	$E'/1{,}000$	L	I	$j^2 = \frac{E'I}{P}$	j	L/j
AB CD	−3,189	23,870	25,022	60	0.02467	193.57	13.913	4.313
BE EC	+2,656	19,880	27,695	60	0.02467	257.24	16.039	3.741
BC	−3,506	18,962	27,920	72	0.03339	265.90	16.306	4.416
AE ED	+1,913	14,319	28,000	72	0.02467	361.09	19.002	3.789
Totals	....	....	....	....	....	....	....	....

Table 1 gives the properties of the tubular members of the truss, the stresses under 1.0119 times the design load, and the stiffness and carry-over factors (far ends fixed), the latter being trigonometric functions of the axial loads and obtainable from tables previously cited.[6] The material is carbon steel (Society of Automotive Engineers (SAE) Specification 1025) with a yield point of 36,000 lb per sq in.; and, in the absence of tangent modulus data, the effective modulus is assumed equal to that obtained from the published value[13] of the ultimate column formula. For values of $\frac{P}{A} \geqq 18{,}000$ lb per sq in., the value is

$$E' = \frac{P}{A}\frac{(36{,}000 - P/A)}{1.172\,\pi^2} \qquad (15)$$

Effective moduli in tension have been assumed the same as in compression. The distribution factors at the joints are calculated from the stiffness by the usual procedure, and are shown in Fig. 5(*b*).

The moment distribution is demonstrated in Table 2. The subvalues of r are $r_{BA} = +0.2796$; $r_{BE} = +0.0012$; and $r_{BC} = +0.5581$. The total r-value is $+0.8389$, which, being less than unity, indicates by the series criterion that the truss is stable under the load factor 1.0119. (It can be shown by repeated trial that the true critical load factor is approximately 1.0309.)

[13] "Airplane Structures," by A. S. Niles and J. S. Newell, John Wiley & Sons, Inc., New York, N. Y., 3d Ed., Vol. II, 1943.

The stiffnesses S' of the members at joint B are computed immediately from Eq. 9. Thus, in inch-pounds,

$$S'_{ba} = 1{,}241.9 - 0.2796 \times 17{,}885.6 = -\ 3{,}758.9$$
$$S'_{be} = 15{,}943.7 - 0.0012 \times 17{,}885.6 = +\ 15{,}922.2$$
$$S'_{bc} = 700.0 - 0.5581 \times 17{,}885.6 = -\ 9{,}282.0$$
$$\Sigma S'_b = \cdots\cdots\cdots\cdots = +\ 2{,}881.3 > 0$$

Only now is the analyst in a position to use the stiffness criterion, although this is obviously not necessary, since the stability has already been checked by the series criterion.

For a simple symmetrical truss of this type, and for most simplified groupings, the stiffnesses of the remaining members can now be obtained merely

TRUSS MEMBERS SUBJECTED TO 1.0119 DESIGN LOAD
INCHES AND POUNDS)

$\frac{S\,L}{E'\,I}$	$\frac{E'\,I}{L}$	S	ΣS_A	ΣS_B	ΣS_E	C	Member
0.12071	10,288	1,241.9	1,241.9	1,241.9	....	6.92975	AB CD
1.40017	11,387	15,943.7	....	15,943.7	31,887.4	0.299572	BE EC
0.05406	12,948	700.0	....	700.0	....	16.1782	BC
1.40912	9,594	13,519.1	13,519.1	....	27,038.2	0.296658	AE ED
....	....	....	14,761.0	17,885.6	58,925.6	....	

by repeated application of Eq. 3, solving either for an unknown S'-value in terms of the spring constant s at the far end, or for an unknown value of s at the far end in terms of the known stiffness S'. The final values are shown in Fig. 5(*c*). Typical computations are now given:

$$S'_{ab} = 1{,}241.9\left(1 - \frac{6.92975^2 \times 1{,}241.9}{6{,}640.2 + 1{,}241.9}\right) = -\ 8{,}154.6 \text{ in-lb}$$

In this calculation 6,640.2 is the spring restraint s_{ba}, acting at end B of member AB. It is the sum of the stiffnesses S' of the two adjacent members BE and BC.

Now consider member AE. From the preceding calculation for S'_{ab}, the value of s_{ae} is equal to $-\ 8{,}154.6$. This value might also be stated as a restraint coefficient as follows:

$$\text{Restraint coefficient} = \left(\frac{s}{E'\,I/L}\right)_{ae} = -\frac{8{,}154.6}{9{,}594} = -\ 0.850$$

The value for S'_{ae} is readily obtained from the following computation, inasmuch as $S'_{ae} = s_{ab}$, the spring restraint acting at end A of member AB′:

$$-\ 3{,}758.9 = 1{,}241.9\left(1 - \frac{6.92975^2 \times 1{,}241.9}{s_{ab} + 1{,}241.9}\right)$$

from which

$$s_{ab} = S'_{ae} = +\ 13{,}568.6$$

and

$$S'_{ca} = 13{,}519.1\left(1 - \frac{0.296658^2 \times 13{,}519.1}{-\ 8{,}154.6 + 13{,}519.1}\right) = +\ 10{,}520.8$$

These computations illustrate the procedure for a simple, symmetrical truss. Final stiffness values and restraint coefficients are shown in Fig. 5(*c*).

For more complicated and extended frameworks, Eqs. 3 and 9 may not suffice to give all the values of S'. In this case one may resort to the final moments in the bars at each joint obtained from the calculations leading to one determination of r, together with Eqs. 9 and 3 or 4. This procedure can be demonstrated on the truss of Fig. 5(*a*) if the symmetry is disregarded and the stiffness values are computed from joint B to joint C, to joint D, and to joint E. From the values of S' at joint B, determined as shown previously,

TABLE 2.—MOMENT DISTRIBUTION FOR LOA

Cycle	Step	JOINT A			JOINT B				JOINT C	
		AB	AE	Unbalanced moment *M*	BA	BE	BC	Unbalanced moment *M*	CB	CE
	$S/(\Sigma S)$.....	0.0841	0.9159		0.0694	0.8914	0.0392		0.0392	0.8914
	C.....	6.92975	0.296658		6.92975	0.299572	16.1782		16.1782	0.299572
1	Balanced ..				−0.0694	−0.8914	−0.0392	+1.0000		
	Carry-over.	−0.4809							−0.6342	
2	Balanced ..	+0.0404	+0.4405	−0.4809					+0.0249	+0.5653
	Carry-over.	0	+0.0182		+0.2803	+0.0217	+0.4022		0	+0.0217
3	Balanced ..	−0.0015	−0.0167	+0.0182				+0.7042	−0.0009	−0.0193
	Carry-over.	0	−0.0204		−0.0106	−0.0243	−0.0138		0	−0.0243
4	Balanced ..	+0.0017	+0.0187	−0.0204				+0.6555	+0.0083	+0.1896
	Carry-over.	0	+0.0067		+0.0119	+0.0080	+0.1349			
	(Cycles 5 through 17 omitted)									
18	Balanced ..	0	0	0				+0.8388	0	+0.0001
	Carry-over.	0	0		0	+0.0001	0		0	0
19	Balanced ..	0	0	0				+0.8389	0	0
Summation.....		−0.4406	+0.4406	0	+0.2796	+0.0012	+0.5581	+0.8389	−0.5997	+0.7858
Unbalanced moments		....	....	....	r_{BA}	r_{BE}	r_{BC}	r	....	....

Eq. 3 gives $S'_{cb} = -\ 9{,}271$. Eq. 3 can also be used to solve for the restraint s_{cb} since the value of S'_{bc} is known. Thus:

$$S'_{bc} = -\ 9{,}282.0 = 700.0\left(1 - \frac{16.1782^2 \times 700.0}{s_{cb} + 700.0}\right)$$

Solving,

$$s_{cb} = +\ 12{,}148.9$$

—but this restraint at joint C on member CB is provided by the sum of the stiffnesses of the members CD and CE. In other words,

$$S'_{cd} + S'_{ce} = +\ 12{,}148.9$$

Inasmuch as a moment coming from joint B to joint C is distributed at joint C to members CD and CE in proportion to their stiffnesses S', the final moments in Table 2 at ends C of CD and CE provide the distribution ratio. Hence:

$$S'_{cd} = +12{,}148.9 \times \frac{-0.1861}{+0.7858 - 0.1861} = -3{,}770$$

$$S'_{ce} = +12{,}148.9 \times \frac{+0.7858}{+0.7858 - 0.1861} = +15{,}919$$

This process could be continued until all the stiffnesses S' are determined. The foregoing values should (and do) check those previously obtained, within the limits of accuracy of the computation used.

LOAD FACTOR 1.0119 (SEE FIG. 5)

C		JOINT D			JOINT E					Cycle
CD	Unbalanced moment M	DC	DE	Unbalanced moment M	EA	EB	EC	ED	Unbalanced moment M	
0.0694 6.92975		0.0841 6.92975	0.9159 0.296658		0.2294 0.296658	0.2706 0.299572	0.2706 0.299572	0.2294 0.296658		... ...
						−0.2670				1
+0.0440	−0.6342				+0.0612	+0.0723	+0.0723	+0.0612	−0.2670	2
0		+0.3050	+0.0182		+0.1307	0	+0.1693	0		
−0.0015	+0.0217	−0.0272	−0.2960	+0.3232	−0.0688	−0.0182	−0.0812	−0.0688	+0.3000	3
−0.1884		−0.0104	−0.0204		−0.0049	0	−0.0058	−0.0878		
−0.0148	−0.2127	+0.0026	+0.0282	−0.0308	+0.0226	+0.0267	+0.0267	+0.0226	−0.0985	4
(Cycles 5 through 17 omitted)										
0	−0.0001	0	0	0	0	0	0	0	0	18
0		0	0		0	0	0	0		
0	0	0	0	0	0	0	0	0	0	19
−0.1861	0	+0.3878	−0.3878	0	+0.1342	−0.26245	+0.23975	−0.1115	0	Σ
....	....	....	....	....	....	....	....	....	....	M

It should be emphasized that the method of the preceding paragraph is largely academic, however, since in practice, even for the simple truss shown in Fig. 5(*a*), one complete moment distribution is too long for the purposes intended, and small simplified groupings are employed. It is not proposed to explore the merits of any particular group arrangements (a comprehensive study of which has been made by A. Slavin,[14] M. ASCE, at New York University, New York, N. Y.), other than to note as an example an arrangement for which the results are available in the foregoing computations.

[14] "Stability Studies of Structural Frames," by A. Slavin, thesis presented to New York University in New York, N. Y., in May, 1948, in partial fulfilment of the requirements for the degree of Doctor of Engineering Science.

If one were interested, say, in the restraints on the bars immediately contiguous to joint B of Fig. 5(*a*), they could be approximated by stopping the moment distribution at the first cycle of carry backs to joint B, giving an approximate series of subvalues of r, equal to +0.2803, +0.0217, and +0.4022 for BA, BE, and BC, respectively (see Table 2)—or a total value of $r = +0.7042$. Physically such a grouping corresponds to fixing the joints twice removed from the joint at which the unit moment is applied (see Fig. 5(*d*)), and such a grouping generally gives a value of the critical load of a steel truss within 1% or 2% of the true value. The corresponding values of the stiffnesses S' and the restraint coefficients $\frac{s}{E'I/L}$ are given in Fig. 5(*d*)).

With the values of the restraints known, it is a simple matter to calculate the reduced length by Eq. 13*b* and Eq. 14 or by Fig. 4. Applying Eq. 13*b* to member AB, for example, and using the exact values of the restraint factors found in Fig. 5(*c*),

$$(+0.645 + 1.319) + \frac{(0.645 \times 1.319)}{\frac{S}{E'I/L}} + \frac{S''}{E'I/L} = 0 \ldots\ldots (16)$$

It is not difficult to determine from tables[6] that Eq. 16 is satisfied by the S-values and S''-values for $L/j = 4.508$. From Eq. 14, $k = \frac{\pi}{4.508} = 0.697$—which is the effective length factor for bar AB with end restraints corresponding to the stresses in Table 1. These stresses are for a loading condition below the buckling load for the truss. If the same end restraints continued to act on AB, and if the effective modulus E' in Table 1 remained unchanged, the critical load for bar AB could be calculated from Eq. 2, and the result would be 3,480 lb. It would be the buckling load for an isolated bar AB with end restraints and E' as indicated. The value has little significance, however, because member AB is a part of the truss of Fig. 5(*a*). As the loads are increased on the truss, the end restraints and the values of E' decrease and the effective length increases. A study of the change in the value of the length factor k for the compression members AB and BC of this particular truss, over a range of multiples of the design loading, is shown in Fig. 6. As the loading increases, the value of k as determined from Eqs. 13*a* and 14 increases. The reduced length $k\,L$ varies from 0.649 L to 0.716 L for member AB, whereas for member BC it varies from 0.632 L to 0.702 L. If the stress (3,247 lb) in member AB, due to the buckling load of the truss, is used in Eq. 2, and that equation is solved for k, it will be found that k for member AB practically checks the value 0.716 (see

TABLE 3.—TYPICAL VALUES OF k AND r

Load factor	RESTRAINT FACTOR[a] k		Moment r
	AB	BC	
0	0.649	0.632	0.0867
0.5	0.652	0.641	0.1593
1.0119	0.697	0.694	0.8389
1.0310	0.716	0.702	1.0061

[a] Computed from Eqs. 13*b* and 14.

Table 3). On the other hand, a similar calculation for member BC gives a value of k slightly greater than the value 0.702.

This fact would seem to indicate that member AB is the first member of the truss to fail and that the failure of member BC follows immediately thereafter. Actually, all the truss members, tension members as well as compression members, reach neutral equilibrium simultaneously, even though one member may be weaker than other members when considered alone. An extremely precise calculation of end restraints, effective lengths, and instability loads would confirm this conclusion. In fact, the phenomenon of truss buckling is a beautiful illustration to support the theory of limit design. A truss with rigid connections does not fail as a whole by primary instability until every reserve of that element of strength associated with bending resistance is drained from the stronger members to assist the weaker members.

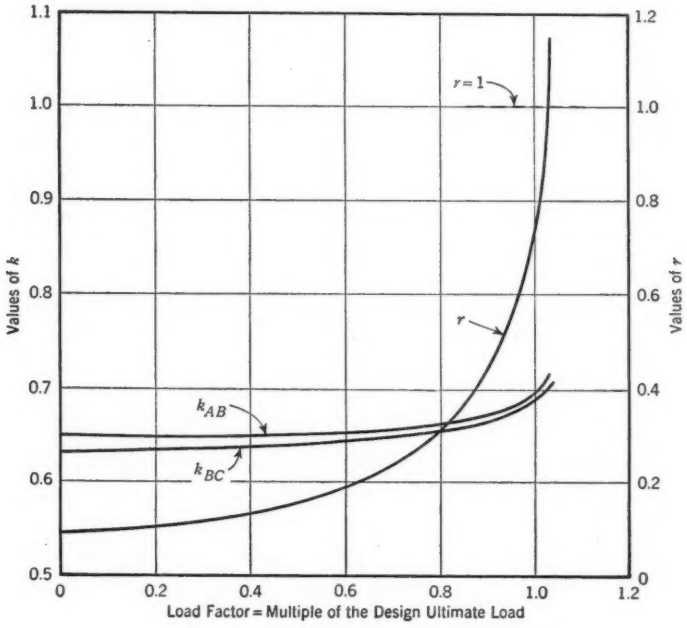

FIG. 6.—PLOT OF k AND r VERSUS LOAD FACTOR

As an illustration, the following theorem relating to the stiffness criterion may be postulated:

> Theorem 1. If the total stiffness of any joint b of a truss becomes zero ($\Sigma S'_b = 0$) under a given loading condition, then the total stiffness for any other joint c also becomes zero ($\Sigma S'_c = 0$) under the same loading condition.

This theorem is intuitively true because, if $\Sigma S'_b = 0$ (that is, joint b is unstable), and $\Sigma S'_c > 0$ (that is, joint c is stable), joint c would have an

unused reserve of stiffness which it would then dispatch to joint b through the medium of the truss framework to assist the latter joint and thus to prevent its instability. Mathematically, the theorem is proved as follows: Referring to Fig. 1(*b*) and applying Eq. 5 to end b:

$$\Sigma S'_b = s_b + S'_{bc} = 0 \quad \ldots\ldots (17a)$$

Hence,

$$s_b = -S'_{bc} \quad \ldots\ldots (17b)$$

—but, from Eq. 3,

$$S'_{bc} = S_{bc}\left(1 - \frac{C^2_{bc} S_{bc}}{s_c + S_{bc}}\right) \quad \ldots\ldots (18a)$$

and

$$S'_{cb} = S_{bc}\left(1 - \frac{C^2_{bc} S_{bc}}{s_b + S_{bc}}\right) \quad \ldots\ldots (18b)$$

Eliminating s_b and S'_{bc} from Eqs. 17*b* and 18,

$$S'_{cb} = -s_c \quad \ldots\ldots (19a)$$

Consequently,

$$\Sigma S'_c = 0 \quad \ldots\ldots (19b)$$

Thus instability at one joint is accompanied by instability at the neighboring joints, and so on through all the joints of the truss. This statement leads directly to the statement of Theorem 2:

> Theorem 2. When a truss fails by instability, all members fail simultaneously, whether they are compression members, axially unloaded members, or tension members.

Intuition again leads to this theorem because, if only one member has a tendency to "fail" in the sense that, considered as an isolated bar with known end restraints, it satisfied Eq. 13*a* whereas other members of the truss still remained stable, the latter would possess a reserve of stiffness which they could dispatch through the framework to the joints at the ends of the distressed member. Only when every last bit of this reserve has been exhausted will collapse occur. All bars will fail at once, because each individually will have no further reserve.

Mathematically, the proof of this theorem is contained in Eq. 13*a*, which was derived from the equation $\Sigma S' = 0$. If the truss is unstable, $\Sigma S' = 0$ at every joint (theorem 1), and Eq. 13*a* is satisfied by every bar entering into every joint.

Theorem 2 introduces the new and perhaps startling concept that an axially unloaded bar in a rigidly connected truss, or a bar with an axial tensile load, can be unstable. Eq. 13*a*, which was set up in a perfectly general way irrespective of the presence of tensile or compressive stresses, applies for all types of members (with the understanding that different tabular values of S and S'' apply for tensile and compressive stresses). For tensile, or axially unloaded bars, S and S'' are always positive; Eq. 13*a* will thus never be satisfied if the restraints s are both positive, as is self-evident; hence, tensile or axially unloaded

bars in a truss which is on the point of buckling are actually subjected to negative end restraints with values that satisfy Eq. 13*a*.

Thus the meaning of the instability of an unloaded bar or a tensile member is clear. A tensile or axially unloaded bar will become unstable if negative restraints exist at one or both ends, with magnitudes such as to satisfy Eq. 13*a*. Generally the presence of restraint increases stability. Conversely, absence of restraint decreases stability. There is no physical reason why the lower limit of restraint must be zero; it can also extend into the negative domain.

If the foregoing concept is understood, the often-appearing statement to the effect that the entire truss fails at once is given powerful meaning and corroboration, because every joint tends toward infinite rotation, and every member (considered as an isolated bar with its appropriate end restraints) becomes unstable. From this point of view it is then incorrect of course to state that one member fails first by primary instability, regardless of whether or not it has been underdesigned. It must be kept in mind that secondary, or localized failure is not being discussed. Actually, such failure would generally occur before the truss failed as a whole.

This "one hoss shay" action of a truss at failure due to primary instability takes place regardless of the make-up of the members, and consequently does not constitute a criterion for economy of the design. There is no question but that the individual members of a truss do contribute in different degrees toward producing failure. Any compression member that would fail if considered pin ended must draw restraint from the adjacent bars; a weakly designed compression member might indeed tend to produce failure by draining off the restraints from an otherwise well-designed framework. Conversely, diagonal compression members designed for part-span loadings help to forestall early failure under full-span loadings, by virtue of their added stiffness under the full-span loadings.

Since the equivalence of the series criterion and the stiffness criterion for stability has already been established, a corollary to theorem 1 may be of some interest:

> Theorem 3. If the series factor, r, becomes 1 for a unit moment applied at any joint b of a truss under a given external load system, r would also equal 1 if the unit moment were applied at any other joint c of the truss under the same external load system.

A graph of r versus the load factor for a unit moment applied at joint B of the truss in Fig. 5(*a*) is shown in Fig. 6. At the critical load factor of 1.0309, $r = 1$. If the unit moment had been applied at joint E instead, r would again reach the value of 1 at the load factor 1.0309. For load factors below this value, however, the r-curve would have different values from the curve shown in Fig. 6. As a matter of practical interest it is to be noted that the joint with the largest r-value for any load factor tends to be the "weakest" joint in the sense that, according to Eq. 10, it is losing its stiffness at the fastest rate.

It is not important to compute effective lengths for any load condition other than the critical load for a truss. Such a procedure was followed in this paper to illustrate one method of calculation and, what is more significant, to

show how the effective length varies as the loading is increased. If one has determined the buckling load for a truss, there is absolutely no necessity for calculating end restraints and then applying Eqs. 13*b* and 14, or for using curves such as those in Fig. 4 to obtain the length factor k. In the light of theorem 2, the Euler equation with the reduced modulus, Eq. 2, is sufficient, and much easier to apply, in securing the value of k for each compression member in the truss. Even if approximate methods are used in obtaining the critical load (methods[14] employing small groups of truss members, in conjunction with the Lundquist-Southwell[15] procedure of extrapolation), the values of the effective lengths at failure may be determined closely enough for all practical purposes from Eq. 2.

The investigation of approximate methods for determining truss buckling loads is beyond the scope of this paper. It will be of some interest however to know that additional investigation has demonstrated clearly that the buckling loads for steel building and bridge trusses, with members having slenderness ratios in accord with current design practice, are so close to the loads corresponding to the yield stress of the steel employed, that there is absolutely no need for buckling load analyses. The yield strength is the ceiling regardless of whether ordinary structural steel, low alloy steel, or silicon steel is used. This conclusion is not true, however, for structural aluminum alloys.

The development of economical design procedures, based on current knowledge of buckling phenomena, is also a matter that cannot be discussed in detail in this paper. It is sufficient to state that design procedures should reflect conditions that exist at ultimate loads, rather than conditions existing at working loads. Otherwise, factors of safety may be quite misleading.

The method presented in this paper is not the only procedure for evaluating end restraints. Classical methods, such as those of H. Zimmermann[9] (who first introduced the idea of spring constants or "Spannziffer") and W. Prager,[16] involve the solution of simultaneous equations containing the restraints as unknowns. An iteration procedure devised by P. T. Hsu[17] involves the assumption and successive refinement of end restraints, and the results in this paper check those obtained on the same problem by Mr. Hsu. A complete historical statement of the various methods relating to restraints and stability analysis of trusses has been compiled.[18]

As presented in this paper, the method is applicable only to trusses, where the effect of joint translation is small. It should be noted that the possibility of appreciable joint translation such as would exist in rigid frames, not only complicates the analysis but widens the range of possible values of k. Although the lower limit of k remains at 0.5 (fixed-ended column), the upper limit is increased to $k = 2$ (cantilevered column). The range of loads at which buckling may take place may be increased considerably.

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[15] "A Method for Estimating the Critical Buckling Load for Structural Members," by E. E. Lundquist, *Technical Note No. 717*, National Advisory Committee for Aeronautics, Washington, D. C., July, 1939.

[16] "The Buckling of an Elastically Encastred Strut," by W. Prager, *Journal*, Royal Aeronautical Soc., November, 1936, p. 833.

[17] "Elastic Stability of Members in Trusses," by P. T. Hsu, thesis presented to Cornell University, Ithaca, N. Y., in May, 1947, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

[18] "Instability of Plane Truss Frameworks," by T. C. Kavanagh, thesis presented to New York University, New York, N. Y., in April, 1948, in partial fulfilment of the requirements for the degree of Doctor of Engineering Science.

Acknowledgments

The paper is part of a series of studies of the behavior of compression members made while both writers were at New York University.

Appendix

B. W. James[7] has demonstrated that, for a prismatic bar of length L, moment of inertia I (about the centroidal axis normal to the plane of bending), and effective modulus of elasticity E', and acted on by an axial load P, the stiffness and carry-over factor are as follows:

Carry-over factor—

$$C = \frac{\alpha}{2\beta} \quad \text{(20a)}$$

Stiffness, far end pinned—

$$S'' = \frac{3}{4\beta} \quad \text{(20b)}$$

Stiffness, far end fixed—

$$S = \frac{3\beta}{4\beta^2 - \alpha^2} \quad \text{(20c)}$$

in which, for compressive loads P,

$$\alpha = 6\frac{\frac{L}{j}\csc\frac{L}{j} - 1}{\left(\frac{L}{j}\right)^2} \quad \text{(21a)}$$

and

$$\beta = 3\frac{1 - \frac{L}{j}\cot\frac{L}{j}}{\left(\frac{L}{j}\right)^2} \quad \text{(21b)}$$

whereas, for tensile loads P,

$$\alpha = 6\frac{\frac{L}{j}\operatorname{csch}\frac{L}{j} - 1}{-\left(\frac{L}{j}\right)^2} \quad \text{(22a)}$$

and

$$\beta = 3\frac{1 - \frac{L}{j}\coth\frac{L}{j}}{-\left(\frac{L}{j}\right)^2} \quad \text{(22b)}$$

For both tensile and compressive loads P,

$$\frac{L}{j} = \frac{L}{\sqrt{\frac{E'I}{|P|}}} \quad \text{(23)}$$

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

WATER SUPPLY ENGINEERING

REPORT OF THE COMMITTEE ON WATER SUPPLY ENGINEERING OF THE SANITARY ENGINEERING DIVISION FOR THE TWO YEARS ENDING DECEMBER 31, 1948

During the years 1947 and 1948 those in charge of water supplies both public and private have sought despite scarcity of materials, slow deliveries, and high and rising prices to correct deficiencies in supply which accumulated during World War II and were accentuated by the postwar increases in demand. These increases were a surprise to most water works officials and engineers.

The increase in domestic consumption was expected. This requires many new services and extensions of small mains but does not count up too rapidly in load on the sources of supply and transmission systems. What has caused inadequacy of many supplies and transmission systems is the unexpected increase in industrial load caused by continued demand in the United States, augmented by government aid to foreign countries and a recrudescence of demand for war goods due to the so-called "cold war" waged by Russia and its old and new satellites. An accumulation of deficiencies left over from the economic depression and not attended to during World War II—partly because mutual aid was arranged for between adjacent supplies and in some cases also because an excess of supply to satisfy lawn sprinkling requirements in periods of peak demand was not regarded as a necessity in wartime—has contributed to the present need.

Cast iron pipe continues to be hard to get with some foundries sold out a year ahead, but there has been some recent improvement. Reinforced concrete pipe, although it has risen in price together with other things, has continued to be used extensively for larger mains; deliveries, or costs, or both, were in such cases more favorable than for metal pipes. Large mains for San Francisco, Calif., and for the East Bay Municipal District were notable exceptions where cement-mortar-lined steel pipe was obtained on more favorable terms.

Pumping units and electric motors continue to rise in price but deliveries are a little better—4 months or so for smaller units and 6 months or 8 months for larger units. Electric motors and control equipment are likely to be slower in delivery than the pumps themselves.

Steel tanks are high in cost with completed tanks delivered in about 15 months; and, in at least some instances, tank manufacturers refuse to under-

NOTE.—Please forward all comments on this report directly to Chairman Thomas H. Wiggin, 90 Broad Street, New York 4, N. Y.

take the painting because of labor union difficulties. Pre-stressed concrete tanks with dome roofs and floors of gunite and walls of either poured concrete or gunite are being used rather extensively. The storage tank at ground level with pumps is being selected more frequently in place of elevated tanks, partly because pre-stressed concrete tanks can be obtained more promptly and partly because there is a growing realization of the difficulty in filling elevated tanks during extreme peaks, particularly where several tanks are on one service level.

Interest rates, especially for municipal bonds, continue to be low, although there is a tendency toward increasing them. The condition favors municipal construction; but against that are the needs for increasing salaries and the higher cost of supplies. High taxes take such a large share of income that there is considerable resistance on the part of voters to approve bond issues for new work. Consequently progress even in municipal work is relatively slow. Privately owned water supply utilities are making still less progress because higher wages and other operating costs have not been covered by increased rates in most cases and capital is not easily attracted. Federal works, largely under the United States Engineer Corps, continue to boom.

Important Water Works Construction in Various Parts of the United States

Massachusetts Metropolitan District Commission.—The easterly part of Hultman Aqueduct is under construction. This aqueduct extends 23.8 miles from the terminal chamber of the Wachusett Aqueduct to the Chestnut Hill area in Boston. It will be recalled that Wachusett Aqueduct comes from the 63-billion-gal Wachusett Reservoir, in itself an important source of water, and now receives the additional supply from the huge new Quabbin Reservoir on the Swift River and from an intake on the Ware River, both farther west. The purpose of the Hultman Aqueduct is to reinforce the old grade-line aqueduct system, to eliminate certain sources of pollution, and to bring water from the Wachusett system under pressure all the way to Chestnut Hill with a loss of only about 8 ft or 10 ft from the hydraulic grade at the terminal chamber. Under the past method of operation, water from Wachusett Reservoir flowed through the Wachusett Aqueduct to Sudbury Reservoir, thence through the Sudbury and Weston aqueducts to the Chestnut Hill Pumping Station and the Weston Distributing Reservoir, respectively. The new work when completed will make it possible to eliminate the present necessity of pumping at Chestnut Hill to the Southern High Service.

The west section of the Hultman Pressure Aqueduct beginning at the equalizing chamber at the end of the Wachusett Aqueduct, a grade-line conduit, consists of about 2 miles of 12-ft 6-in. reinforced concrete pipe built by cut-and-cover methods, then the 14-ft 0-in. tunnel about 3 miles long under Sudbury Reservoir and adjacent high ground, and finally the 11-ft 6-in. cut-and-cover reinforced concrete pipe which ends at downtake shaft No. 5, just west of the Charles River, making a total length of 18.3 miles. This part of the Hultman Aqueduct was built from 1938 to 1940. From shaft No. 5 east the aqueduct is referred to as the City Tunnel. Construction on the project was resumed in 1946 by awarding a contract for two of the shafts in the City Tunnel section—

shaft No. 6 at about the halfway point in Newton and shaft No. 7 near the Chestnut Hill end. Work on the tunnel proper is now under way.

The tunnel between shafts Nos. 5 and 6 is 2.26 miles long and 12 ft in diameter; between shafts Nos. 6 and 7, it is 2.53 miles long and 12 ft in diameter; and between shafts Nos. 7 and 7B, it is 0.65 miles long and 10 ft in diameter. Excellent rock conditions have prevailed thus far (October, 1948).

The shaft structures will provide means for connecting the tunnel with various parts of the distributing supply lines.

Shaft No. 6 is on a large vacant lot owned by a cemetery corporation. The land is low and swampy. An agreement has been reached to use tunnel spoil to fill in the low areas. The contractor is required to see that no tunnel spoil containing stones larger than 4 in. in size is deposited within 5 ft of the final surface, and the surface of the spoil is to be covered with soil stripped from the area and graded to form an attractive topography. Tunnel progress to October 1, 1948, is given in Table 1.

TABLE 1.—TUNNEL PROGRESS TO OCTOBER 1, 1948

Description	Shaft No. 6	Shaft No. 7	Shaft No. 7
Finished tunnel; size, in feet	12	12	10
Weekly Progress; Tunnel Excavation per Heading, in Feet:			
Average in most recent period of 6 months	170	160	182
Recent maximum	210	191	240

The district has also begun construction of a feeder line from Quabbin Reservoir southward to serve the communities of Ware, Palmer, and Chicopee. About 15 miles of pipe lines varying from 48 in. to 16 in. in size is being constructed. In accordance with the action of the 1945 legislature the price of water from Quabbin Reservoir is fixed at $20 per million gal at the reservoir. Capital charges incurred by the extensions plus operation and maintenance will raise the cost of water delivered to the respective town lines to from $60 to $65 per million gal.

Springfield, Mass.—A second steel pipe line 48 in. in diameter extending from the West Parish filters to the Provin Mountain Reservoir is being completed.

Board of Water Supply, New York, N. Y.—Work continues actively on the additional supply from Rondout Creek, Neversink River, and East Branch of the Delaware River which together will add a safe, dry weather yield estimated at 540,000,000 gal daily. Consumption of water has increased greatly in greater New York, having reached more than 1,300 mgd last summer with a safe yield from upland and ground-water sources of only about 1,000 mgd. It is a race between the incidence of a very dry year or series of dry years and the completion of the new construction.

Works under way include equipment for the Delaware Aqueduct between Rondout Reservoir and West Branch Reservoir of the Croton system and thence to Kensico and Hillview Reservoir; completion of Merriman Dam on Rondout Creek (estimated yield 100 mgd to be finished in about a year—natural flow of the stream already in use for several years); and caisson core

walls for Neversink Reservoir and for East Branch Reservoir. The contract for the aqueduct to convey the East Branch and Neversink Reservoir supplies is still to be let. The time required to complete the whole Rondout, Neversink, and East Branch development estimated to give an additional safe yield of 540,000,000 gal daily will be from 7 to 8 years more.

Early in 1948 the Board of Water Supply appointed a board of three engineers to study possible emergency supplies for New York City pending the completion of additional supplies from the Delaware River sources.

Metering in New York City.—A recrudescence of the often-discussed project of general metering has occurred and Mayor William O'Dwyer's Executive Committee on Administration reported in favor of metering all multiple-family dwellings housing three or more families. This program was to cover a period of 5 years at a cost of $30,000,000, cost to be refunded to the city by property owners over a period of 10 years. The work of metering, if done, would come under the Department of Water Supply, Gas and Electricity which operates all the city's water supplies, including the large upland supplies constructed by the Board of Water Supply after they are completed and turned over.

The Philadelphia, Pa., Water Works Improvement.—A board of engineers recommended a new 500-mgd filtered supply from the Delaware River taken at a proposed dam to be located at Wallpack Bend, about 100 miles above Philadelphia, brought by a pressure tunnel 15 ft 6 in. in diameter to a reservoir of 22-billion-gal capacity, purified in modern rapid sand filters including of existing filters only the Queens Lane Plant, and delivered through a greatly strengthened distribution system. The city authorities decided instead to improve the present sources.

A bond issue of $18,000,000 had been approved by the electorate of the city for a water works improvement program in April, 1940. This program has been put into effect. The improvements completed and under contract totaling this amount consist of the following:

1. A new filtered water pumping station at Torresdale near the principal slow sand filter plant on the Delaware River with electric motor-driven pumping equipment having a capacity of 215 mgd; new electric motor-driven pumping equipment, having a capacity of 210 mgd, to replace obsolete steam pumps and boilers at the Lardners Point Filtered Water Pumping Station; new electric motor-driven pumping equipment, having a capacity of 200 mgd for the Queen Lane Raw Water Pumping Station, to replace obsolete steam equipment; new high service booster station with electric motor-driven pumping equipment for the Fox Chase District; and two 48-in. force mains between the Belmont Raw Water Pumping Station and Sedimentation Reservoir.

2. An ozonation system at the Belmont Filter Plant, having a water treating capacity of 36 mgd; prechlorination, postchlorination, and chemical feeding equipment at the Torresdale Filter Plant; rehabilitation of a part of the Torresdale Slow Sand Filters; prechlorination, postchlorination, and chemical feeding equipment at the Queen Lane Filter Plant; a new flume at the Queen Lane Raw Sedimentation Reservoir; new heating systems at Queen Lane and Bel-

mont filter plants; general rehabilitation of rapid sand filters at Queen Lane and Belmont and of prefilters at Torresdale and Queen Lane filter plants; and cleaning of the Torresdale, Belmont, Queen Lane, and Roxborough raw water reservoirs by hydraulic dredging equipment.

3. Cleaning and repair of the Fairhill Reservoir in the high pressure fire service system; chlorination facilities and equipment and new screens at the East Park, Roxborough, and Oak Lane filtered water reservoirs; and

4. Rehabilitation of the distribution system through the installation of more than 96 miles of force mains and distribution mains from 4 in. to 60 in. in diameter.

The following improvements included in the general program, but not under contract, are estimated to cost an additional $35,000,000 at present prices.

a. Completion of the electrification of the Lardners Point Filtered Water Pumping Station and construction of a new force main system adjacent thereto; remodeling of building and installation of the new electric motor-driven pumping equipment at the Queen Lane Raw Water Pumping Station; installation of new electric motor-driven pumping equipment and rehabilitation of the Belmont Raw Water Pumping Station; provision of new electric motor-driven pumping equipment at the Fairhill and Race Street high pressure fire service stations and of a new intake at Race Street; and construction of new booster stations and pumping equipment at Roxborough, Oak Lane, and Belmont;

b. Construction of a new preliminary treatment plant and reconstruction of the rapid sand filters having a rated capacity of 120 mgd at Queen Lane; reconstruction of roof over the Queen Lane slow sand filters; construction of a complete new rapid sand filter plant, having a rated capacity of 360 mgd and new raw water force mains from the raw water pumping station at Torresdale; construction of a new preliminary treatment plant, having a rated capacity of 72 mgd and new rapid sand filters, having a rated capacity of 32 mgd at Belmont; and

c. Construction of new covered reservoir at Georges Hill and elevated storage facilities in the Fox Chase, Belmont, and Oak Lane districts, and of additional force mains and distribution mains estimated to cost $7,500,000.

Construction plans and specifications for some of these improvements are in progress or have been completed and will be ready for advertising for bids when funds are available.

Baltimore, Md.—The shafts and headings for the 10-ft and 7-ft-diameter Liberty Road-Montebello tunnel from the north branch of the Patapsco River were completed and contracts for the first 7 miles of the 16½-mile tunnel were let in 1947. By June 1, 1948, 25,000 ft of tunnel had been completed.

Washington, D. C.—The District of Columbia started a long-term construction program in 1947. The 1947–1950 phase of the program involves pure water reservoir capacity at the Dalecarlia filter plant and extensive main extensions.

Syracuse, N. Y., City Supply.—In 1948, 4,275 ft of 54-in. steel pipe was added to the Lake Skaneateles transmission main. This construction extends to 12 miles, a 54-in. main paralleling two existing 30-in. mains 19 miles long.

Syracuse Suburban Water Supply.—A new 42-in. concrete intake paralleling a part of the existing intake is about two thirds completed in Otisco Lake. Increased pumping capacity at the lake and a booster station along the transmission main, also about 5,100 ft of 30-in. reinforced concrete parallel main added further along, will increase the supply from 11 mgd to 13.5 mgd to meet steadily increasing industrial and domestic demand. An increasing number of diesel locomotives are reducing the water demands of the New York Central Railroad but other users are more than compensating.

Rochester, N. Y., City Supply.—Canadice Lake, which with Hemlock Lake is the source of water, has been raised and plans are in process for building a new and higher dam at Hemlock Lake. Two of the three conduits which transport this water have been cement lined (sizes, 39 in. and 36 in.–24 in.) and the third conduit, 37 in. in diameter, is under contract for cement lining. These conduits to the first distributing reservoir (Rush) are about 93,000 ft long.

Supply for Suburban Buffalo, (N. Y.).—A considerable and steady growth both in domestic and industrial consumption of water has taken place in suburban Buffalo. Both Erie and Niagara counties are making studies for an increased water supply. The Town of Tonawanda has a state-approved plan for taking a supply from the West Branch of the Niagara River at Grand Island not far downstream from Buffalo but away from its pollution which goes down the East Branch. The Western New York Water Company, which serves the territory bordering on Buffalo, also has submitted a comprehensive plan to the Public Service Commission.

Cleveland, Ohio.—This city is installing a new intake into Lake Erie. It is 18,500 ft long and composed of concrete pipe made in sections 10 ft in interior diameter, with a 10-in. wall thickness, 24 ft long, and weighing 50 tons each. This intake will connect at a shaft with a tunnel 1½ miles long leading to the proposed 150-mgd Nottingham Filter Plant.

Detroit, Mich.—The Detroit Board of Water Commissioners has determined on a 10-year $35,000,000 program of expansion. Rates have been raised 14¢ per 1,000 cu ft—9% for smallest domestic consumers and 29% for largest industrial consumers. Present rates are 78¢ per 1,000 cu ft for the first 10,000 cu ft and 28¢ per 1,000 cu ft for more than 10,000 cu ft. Consumption per capita increased from 130 gal per day in 1942 to 144 gal per day in 1947. The average consumption in 1948 was 250 mgd.; the record day occurring in August, 1942, was 572 mgd; and the peak hour rate was reported as 800 mgd. This peak hour rate approaches in ratio to average consumption, the peak hour ratio of the warmer cities of California, such as Bakersfield and Fresno.

Saginaw-Midland Michigan Water Supply System.—This joint project for delivering 43 mgd with ultimate capacity up to 100 mgd, to supply Saginaw, Midland, and the Dow Chemical Company was described in the committee's report for 1945–1946. It has been placed in operation.

La Fourche Bayou Supply, Louisiana.—A public water supply to serve about 20,000 persons in a rural area running for 57 miles along this bayou which extends from the Mississippi River to the Gulf of Mexico involves 63,000 ft of 14-in. cast iron pipe from the river and 147 miles in two 14-in. cast iron mains, all tar coated.

Summary of West Coast Water Supply Development in 1947–1948.—During World War II and largely because of it, an extraordinary increase in population was experienced on the west coast. California, Oregon, and Washington have increased their population by 45%, from 9,733,000 in 1940 to 14,144,000 in 1948. This great growth, coupled with a series of dry years, dwindling ground-water supplies, and increased industrial demand for water, has placed a tremendous strain on existing water supply facilities and has given rise to a truly immense program of planning and development of new facilities.

Among the new large municipal water supply developments are those of the Metropolitan Water District of Southern California, the East Bay Municipal Utility District, and the cities of San Francisco, Los Angeles, and San Diego, Calif., and Seattle, Wash. The Colorado River Aqueduct was extended to San Diego with the aid of the United States Navy, and has increased that city's supply more than 150%. Arrangements have also been made to extend the Colorado River supply to several smaller cities in the Los Angeles area. Other states participating in the supply from the Colorado River are reported as being concerned about the apportionment of the Colorado supply which is not unlimited.

The East Bay Municipal Utility District and the City of San Francisco are constructing duplicate aqueducts. Enlargement of filtration and distribution facilities is planned for Sacramento, Calif. Construction of distribution reservoirs and extension of large mains are in progress at Seattle.

The Corps of Engineers, Department of the Army, and the Bureau of Reclamation are active in planning and constructing a wide variety of water supply projects, of which many are of the multi-purpose type, to control floods, and to conserve water for irrigation, navigation, power generation, and municipal supply. Many of these projects have become urgent because of diminishing ground-water supplies used for irrigation. The Bureau of Reclamation is proceeding with construction of Davis Dam on the Colorado River, and is active in completion of the Central Valley Project.

A more detailed list of west coast water supply activities follows:

The South Pacific Division of the United States Corps of Engineers.—Construction is reported in progress on two large reservoirs for flood control and regulation of stream flow for irrigation and power as follows: The Pine Flat Reservoir on Kings River in California, with a capacity of 330,000 million gal (construction starting in April, 1947); and Isabella Reservoir on the Kern River, with a capacity of 180,000 million gal (construction starting in March, 1948).

Two other reservoirs are planned with construction beginning in the near future, as funds are in hand, as follows: Folsom Reservoir on the American River in California, with a capacity of 290,000 million gal; and Cherry Valley Reservoir on Tuolumne River.

The United States Army Engineers also are planning work authorized by Congress on six other reservoirs and allied work. Three of these reservoirs are for flood control and have a total capacity of about 330,000 million gal. Three other multiple-purpose reservoirs have a combined total capacity of about 450,000 million gal.

The Corps of Engineers has completed two dams, has two others under construction, and is about to advertise a fifth for bids in Oregon. These dams are a part of the flood control project for the Willamette River Basin and will also provide irrigation, navigation, power, and stream purification benefits.

It may help in realizing the extent of reservoir construction done and to be done by the Corps of Engineers largely under a new law described in the report of the committee for 1945–1946, to note a recent statement of the Chief of Engineers that the combined area of these reservoirs is about 1,500,000 acres, or more than 2,300 sq miles—practically the area of the State of Rhode Island.

The Bureau of Reclamation.—Region II reports a large amount of work under way consisting mainly of the Central Valley Project, principal supply for which is from Shasta Dam on the Sacramento River. The work on this project included installation of drum gates to increase the storage at Shasta and Friant dams, and construction of Keswick Dam, Tracy Pumping Plant, and Delta Mendota Canal. Various irrigation projects are being planned for Arizona. The Coachella branch of the All-American Canal has been completed.

Two proposed projects are the Solano County Project and the Cachuma Unit, Santa Barbara Project, involving reservoirs having a combined capacity of 600,000 million gal. The over-all cost of the two projects is estimated to be about $80,000,000. In addition, there are three irrigation district projects estimated to cost about $27,000,000.

Region III, which embraces Southern California, Arizona, and Nevada, reports progress on the Davis Dam on the Colorado River, together with a number of other projects for irrigation and electric power construction.

Public water supply systems on the west coast have generally experienced substantial increases in water demand since the end of World War II with the resultant need of expanding supply and distribution systems.

San Francisco.—To provide for the increase in demand in California, the city awarded contracts in the summer of 1948 for the construction of some 20 miles of the Joaquin pipe line No. 2 of the Hetch Hetchy Aqueduct. This pipe line is to be 61 in. in inside diameter, and 47.5 miles long, with a flow capacity of 75 mgd.

Bids for two sections of this line were called for with alternate bids for steel plate pipe and reinforced concrete pipe. Steel plate pipe, 62 in. in inside diameter and $\frac{1}{2}$ in. thick, with cement mortar lining and outside coating, is to be installed for 21.5 miles at a cost of about $5,300,000. For 10.8 miles, the contract was made for lock-joint steel cylinder reinforced concrete pipe, 61 in. in inside diameter, to cost about $2,180,000. These two sections will increase the delivery of the aqueduct from 62 mgd to 87 mgd. The two sections will be completed by about October 1, 1949.

San Francisco also has a quantity of distribution pipe line and pumping station work under way.

Sacramento.—This city is in need of a major addition to its filtration plant.

Seattle.—Seattle has been carrying out reinforcements to its distribution system, including large pipe and elevated storage, at a cost of more than $2,000,000.

East Bay Municipal Utility District.—A major construction item for this district which supplies Oakland and adjacent municipalities across the bay from San Francisco is the construction of a second pipe line on the Mokelumne Aqueduct, which is to cover a distance of 81 miles at an estimated cost of $22,000,000. The major contracts have been awarded and construction has been carried on during the past 2 years. The project is about 75% completed. The upper 15 miles of the line is made up of steel cylinder pre-cast concrete pipe in 24-ft lengths. The remainder of the line is made up principally of 68-in. steel pipe with plate thicknesses ranging from $\frac{3}{8}$ in. to $\frac{1}{2}$ in.

The capacity of the present aqueduct is 95 mgd if two booster pumping plants are used. The new line is to have a capacity of approximately 50 mgd by gravity, which can be increased to slightly more than 100 mgd by the installation of booster plants.

Cathodic protection has been continued along the Mokelumne Aqueduct by the operation of eighteen stations at an annual cost of approximately $15,000. These stations are reported to give complete protection to 81 miles of steel aqueduct consisting mainly of 65-in. steel pipe.

In the East Bay District, the consumption in 1941 was 46 mgd, but by 1946 it had increased to 106 mgd, remaining practically stationary for the last 2 years.

Los Angeles.—The total supply of 340 mgd available from the Los Angeles system is now consumed. Plans have been developed by Los Angeles for the major distribution trunk lines to deliver the new supply. Additional water to increase the total supply to about 580 mgd is to come from the Colorado River system of the Southern Metropolitan Water District Commission. It is estimated that the total supply will provide for 4,000,000 people—double the present 2,000,000 population of Los Angeles.

Los Angeles is also constructing a new 300-million-gal distribution reservoir and is actively carrying on the installation of various trunk mains that are from 45 in. to 66 in. in diameter.

The Metropolitan Water District of Southern California.—This district has continued to add districts to its area with four cities joining during 1948. Construction on an additional 100-mgd softening and filtration plant—doubling the capacity of the treatment plant for the Colorado River water—was started in February, 1948, and is scheduled to be completed in mid-1949.

San Diego.—This city is now receiving Colorado River water from The Metropolitan Water District of Southern California through the 71-mile, San Diego Aqueduct which was completed in November, 1947. The new aqueduct provides about 67 mgd and 85% of this amount is for the City of San Diego. A 66-mgd softening and filter plant is being installed by San Diego at an estimated cost of $2,500,000. This plant is so designed that it may in the future be expanded to 100 mgd. A lime-zeolite softening process is to be used. The San Diego County Water Authority was established to finance the construction of the San Diego Aqueduct and other work necessary to deliver Colorado River

water when a very serious shortage of water supply in the San Diego County area had developed as a result of low rainfall and rapid growth in demand for water supply in this area. The San Diego Aqueduct was constructed to one half of the total capacity except for the tunnels, which represent 4.4 miles. Studies are already being made as to the need and feasibility of increasing the capacity of the San Diego Aqueduct at an early date.

San Juan, Puerto Rico.—Augmented water supply works for San Juan were constructed during the late years of World War II and were completed in 1946. They included a 10,000,000-gal distribution reservoir, 70,000 ft of 36-in. and 42-in. pre-stressed reinforced concrete pipe, a concrete gravity dam 550 ft long with a maximum height of 100 ft, and an earth fill dam 950 ft long with a maximum height of 75 ft. These works at a cost of $5,100,000 increased safe yield of the San Juan supply from 16 mgd to 26 mgd. When these works were started, the water works were owned by the city. In 1945 an act of the legislature created the Puerto Rico Aqueduct and Sewer Service and transferred to it all property and franchises operated or contracted by the municipalities. The San Juan system was expanded to serve eleven instead of four adjoining communities with an estimated demand in 1950 of 38 mgd. To provide this quantity of water a concrete gravity dam 108 ft high on the Rio Grande de Loiza and water treatment facilities have been designed. It is estimated that this project will provide an initial capacity of 30 mgd and an ultimate capacity of 60 mgd.

Notable Floods

The June-July, 1947, flood on the lower Missouri and central Mississippi rivers, was generally considered the worst in more than 100 years.

The annual *Water Resources Review* of the United States Geological Survey for the water year ending September 30, 1947, reported that crest stages on the lower Missouri were only slightly below those of 1844. The damages reported in Iowa, Nebraska, and Missouri were tremendous.

Three extreme floods occurred in Iowa during June and peak flows of the Des Moines River twice exceeded the historic flood of 1903.

Thirteen lives were lost in the flood on Medicine Creek at Cambridge, Nebr., in June. The flow from 800 sq miles of tributary watershed at this point was recorded at about 120 cu ft per sec per sq mile.

In August, 1947, a damaging flood occurred in the Little River Basin in Arkansas and Oklahoma. The peak flow of Mountain Fork and Cossatot rivers equaled or exceeded all peaks since the flood of August, 1915. The flow of Mountain Fork River from 600 sq miles of tributary watershed was about 110 cu ft per sec per sq mile. From the Cossatot River with some 380 sq miles of watershed, the flow was about 120 cu ft per sec per sq mile.

Hurricane floods of September, 1947, which occurred in southern Florida almost completely inundated the Everglades with Lake Okeechobee reaching a new record high level. Heavy overland flow occurred, with extensive flooding of farm lands. High water continued in this area well into 1948.

During the two water years ending in September 30, 1948, the annual flow of all major rivers in the United States and Canada was near or above normal.

In 1947, the runoff of the Missouri River was excessive and the flow of the Saint Lawrence River was near the record high.

In 1948, the most disastrous flood in the history of the Pacific northwest occurred at the end of May and early in June in the Columbia River and Fraser River basins, caused mainly by delayed melting of a heavy accumulation of snow in the mountains. This flood was exceeded by that of 1894 on both streams. Peak discharge for the 34,000 sq miles of Columbia River at Birchbank, B. C., Canada, reached 10.8 cu ft per sec per sq mile and near The Dalles, Ore., with 237,000 sq miles, the discharge reached 4.26 cu ft per sec per sq mile.

Floods in Mississippi, Alabama, and Georgia in November and early December, 1948, were of record-breaking magnitude. The peak flow of Ocmulgee River at Macon, Ga., was 80,000 cu ft per sec exceeding all floods since the record began in 1893. For this 2,240 sq miles, the flow was 35.7 cu ft per sec per sq mile.

Record-breaking floods in late December, 1948, and early January, 1949, followed heavy rainfall of as high as 8 in. to 11 in. in 3 days in eastern New York and western New England. Peak discharges exceeded those of 1913 and 1938 in the upper reaches of the Hudson, Naugatuck, and Housatonic rivers. Damage was extensive and five lives were lost.

Drought Conditions

Beginning in February, 1947, a drought condition began to develop in southern California, New Mexico, and Arizona. This continued unabated through the spring of 1948, with runoff far below normal. The 11 months from January 1 to November 30, 1947, constituted the driest such period on record for southern California since American occupancy according to the November issue of *The Colorado River Aqueduct News*. At Los Angeles the total rainfall for that period was 2.49 in. compared with 2.89 in. in 1894, the lowest previous record.

According to the *Water Resources Review* of the U. S. Geological Survey, there was no flow during either January or February in the Santa Ynez River near Lompoc, Calif., the first time in the 32 years of record that this condition has occurred in either of these months.

In August, 1947, the water levels in the deep artesian wells of Atlantic City, N. J. (depth of some 800 ft), and Ocean City, N. J., were reported lower than they had ever been before.

Ground-Water Conditions During 1947–1948

Pumpage.—Areas of greatest withdrawals are Los Angeles (360 mgd), Long Island, New York (280 mgd), Houston, Tex. (160 mgd), Memphis, Tenn. (105 mgd), and San Antonio, Tex. (100 mgd). States of greatest withdrawal are California (5,625 mgd—chiefly irrigation), Arizona (1,550 mgd—chiefly irrigation), Texas (1,325 mgd—about half irrigation), Louisiana (700 mgd—about half irrigation), and New York (660 mgd—chiefly public supply and industrial). These estimates are based on a canvass made by Geological Survey.

Salt Encroachment.—Problems continue to exist along all coasts. Areas of considerable importance are Long Beach, Calif.; Galveston, Tex.; Miami, Fla.; Savannah, Ga.; Elizabeth City, N. C.; Baltimore, Md.; several areas in New Jersey—notably Atlantic City; Long Island; New Haven, Conn.; and Providence, R. I.

Water Levels.—During the period involved, there was no nation-wide decline of the ground-water level. In many states water levels were at or above average levels. However, progressive declines continued in nearly all heavily pumped areas, Indianapolis, Ind., and Long Island being exceptions. Critically low levels occurred in several areas in Louisiana, California, and Texas, and progressive declines continued in areas in Arizona and New Mexico, and at Pittsburgh, Pa., Memphis and Louisville, Ky. Recovery continued at Indianapolis. Rises since 1944 range from several feet to 20 ft.

Ground-Water Recovery at Flatbush, Kings County, Long Island, N. Y.—Pumping at the rate of approximately 25 mgd from about thirty deep wells distributed over an area of about 6 sq miles in Flatbush ceased abruptly on June 30, 1947, when the City of New York turned city water into the area after condemnation of the private water company which for many years had supplied this territory from wells varying from about 150 ft to 430 ft in depth.

Static levels of ground water were from about 10 ft to 35 ft below sea level and salt-water intrusion had hurt the quality of the water from many of the wells. The U. S. Geological Survey has been using a number of the wells to determine the rate of rise of static level since the cessation of pumping. There are a number of other wells still operating in Flatbush but not of great aggregate capacity. There is also a larger draft from industrial wells north of Flatbush so that the rise in static level is not entirely related to the cessation of the main draft. The Geological Survey reports that the recovery in Flatbush in the 18 months since cessation of use of the wells has ranged from 7 ft to 15 ft and has averaged 10 ft. Measurements of the Geological Survey show that the water table in more than half of Brooklyn now lies above sea level. The rate of recovery is flattening off. Similar recoveries have also occurred in the Jameco artesian beds, with the piezometric surface lying about a foot below the water table. The rate of recovery in the water table is now greater than the rate of recovery of Jameco artesian levels whereas the rate was more or less similar immediately after shutdown.

The Flatbush shutdown has caused rises of water level in wells situated near the center of the water table depression in Brooklyn to the north of the Flatbush area. A diagnostic well shows a 4-ft rise in 18 months and its water level is higher than at any time since 1935 when measurements were started.

Conservation of Ground Water Used in Air Cooling.—For more than 10 years the New York State Water Power and Control Commission has required conservation of ground water from well supplies exceeding about 60 gal per min in capacity used in cooling the air in summer in theaters, stores, etc., to be circulated in tight pipes of noncorrodible metal and returned to the ground. The success of this requirement was not too great at first because of lack of experience in constructing return wells. As a result of the frequent failure of such wells

the rule was often violated and the water sent to the sewer. Gradual improvement in design of the return wells greatly increased the success of the operation and a high and increasing percentage of the cooling water is now being returned to the ground. The temperature of the pumped water has been markedly increased in some cases by heating the ground.

An act which makes provisions similar to those in New York State but applicable to well supplies of more than 200 gal per min in capacity was passed in 1947 in Indiana and became effective on January 1, 1948.

Special Methods of Exploring for Ground Water.—Seismic methods—that is, the timing to one one thousandth of a second of the speed of sound waves induced by an explosion of dynamite—have been employed successfully for a number of years for exploring the depths to rock and the nature of the overlying material for highway, dam, and other foundations. By the use of such methods, strata favorable for ground-water development have been located.

Electrical survey methods have been used in rotated water wells, with results claimed to be very informative where the material stands without clay water; but, without success, as far as the committee knows, where the walls of the well are held up by clay water.

Electrical surveys made without borings are said to have furnished guidance in the selection of territory to explore for well water.

These newer methods have been highly developed in the search for oil and their adaptation to water wells has been begun.

Abatement of Stream Pollution

After many years of wrangling over the terms of a federal water pollution control act, such an act became law by being passed at the close of the Congressional session in June and signed by President Harry S. Truman on June 30, 1948.

The bill recognizes the principle of state control of pollution abatement and authorizes the United States Surgeon General to cooperate with all agencies to prepare or adopt comprehensive programs for eliminating or reducing pollution of interstate waters, where such pollution endangers the health or welfare of persons in another state than the one where the pollution originates. The state health department is recognized as the "state water pollution agency" unless another state agency is provided.

If the pollution of interstate waters is not abated, the federal security administrator is authorized to call a public hearing before a board of five or more persons appointed by him. Such a board is to advise the administrator as to equitable and reasonable abatement measures to be taken. If such measures are not taken within a reasonable time, the administrator, with the consent of the pollution agency of the state in which the pollution is discharged, may request the Attorney General to bring suit on behalf of the United States to secure abatement.

The Federal Works Administrator is authorized to make loans limited to one third of the estimated cost, but in no case exceeding $250,000, to any state, municipality, or interstate agency for reports, plans, and the construction of treatment works for pollution abatement. The projects must have the prior approval of the appropriate state agency and the Surgeon General.

In the Public Health Service, a Water Pollution Control Advisory Board of eleven members is to be made up of five officers of the federal government and six persons who are not officers or employees of the federal government, to be appointed annually by the President of the United States. This board is to review the policies and program of the Public Health Service as undertaken under the authority of the act and to make recommendations thereon in reports to the Surgeon General.

The financial authorizations under the act total $139,500,000 for 5 years. The maximum amounts authorized for each of the 5 years beginning July 1, 1948, are as follows:

To Federal Security Agency—	
For loans	$22,500,000
To be paid to states for surveys, etc.	1,000,000
To carry out its functions	2,000,000
To Federal Works Agency—	
For Public Health Service buildings at Cincinnati, Ohio	800,000
For grants for preliminary studies	1,000,000
To carry out its functions	500,000
Total yearly authorization	$27,800,000

The Public Health Service proposes to divide the United States into river basins and first to improve the conditions of the worst polluted streams.

Ohio River Compact.—In line with the federal act above described, a compact has been made between the states of Indiana, West Virginia, Ohio, New York, Illinois, Kentucky, Pennsylvania, and Virginia for abating pollution in the Ohio River. This agreement is very complete, embracing formation of a commission with a chairman.

A New England Interstate Pollution Control Compact has been signed by representatives of Connecticut, Massachusetts, and Rhode Island, and it is hoped that other interested states, including New York, will join in this movement.

In New York, a Joint Legislative Committee has prepared a proposed water pollution control act for introduction in the 1949 legislature. This act will create a Water Pollution Control Board in the Department of Health, and will provide for classification of the waters of the state.

Pipe Materials

Growth in Percentage of Pipe Made Centrifugally in Metal-Lined Molds.—This growth has been rapid, three additional companies having shifted from other methods.

Protection of Cast Iron Pipe.—Some, but not notable, progress has been made in the education of water supply engineers and officials in the use of cement lining for the interior of cast iron pipe as noted in Table 2, the data for which have been kindly furnished by the foundries.

The committee is still concerned over the high percentage of pipe, probably nearly 50%, with only tar dip coating, and also over the considerable percentage of thin cement lining which considered in terms of decades may not be durable against leaching out to a friable condition which will not prevent tuberculation.

Lining Pipe in Place with Cement Mortar.—During 1947 and 1948 up to October, about 300,000 lin ft of 48-in. to 24-in. cast iron and steel pipe was lined in place with cement mortar by a company which applies premixed mortar centrifugally and smooths it with revolving trowels. About 75,000 additional ft were under contract. The company which applies the lining by cement gun and smooths it with revolving trowels also did a considerable footage of similar work.

TABLE 2.—PERCENTAGE OF CAST IRON PIPE PROTECTED ON THE INSIDE BY VARIOUS METHODS

Foundry	Cement mortar of standard thickness	Percentage of Col. 2 with bituminous coating	Thin cement mortar with bituminous coating	Tar dip
(1)	(2)	(3)	(4)	(5)
A	4.6[a]	0.6	37.0	33.4
	1.0[b]	0.2	53.2	22.2
B	20	98[c]	29	51
C	45	No record	0	41
D	12.5	96	0	87.5
E	40.	100	0	....

[a] 3 in. to 12 in. in diameter. [b] 14 in. to 48 in. in diameter.
[c] Bituminous coating used as a cure aid for cement lining.

The period 1947–1948 was notable because the first company added the 24-in. size to pipe thus lined, nearly closing the gap between the smaller sizes lined by mandrel and the larger sizes lined as described previously.

The mandrel method of lining pipe in place continues to be used in sizes up to and including the 16-in. size. Recent contracts include 18-in. pipe and it is stated that 24-in. pipe has been lined by this method in Australia. During 1947 and 1948 more than 97,000 ft of pipe in the United States was lined by the mandrel method and the total so lined in this country to date is about 900,000 ft.

Lining Pipe in Place with Asphalt.—The process of lining metal pipe in place with a thin coating of asphalt, deposited electrically from an emulsion, has been brought to the United States from England where it has been used to a limited extent for many years. A considerable footage of such lining work has been done during the 2-year period covered by this report.

Cathodic Protection for Exterior of Pipe Lines.—This process is a well-tried and frequently used method of protecting cross-country gas and oil lines. The first Mokelumne pipe line, of steel 72 in. in diameter, and about 80 miles long, for the supply of Oakland and adjacent territory has been successfully protected by the cathodic process for many years. The second Mokelumne pipe line, of steel 67 in. in diameter and 81 miles long, is also being so protected but problems have arisen and are under active study occasioned by some interference between the protective work of the two pipe lines, which apparently requires more electric power per pipe than was needed for the first pipe. The committee has been informed that a pipe line of the San Diego water system has cathodic protection.

Pre-Stressed Concrete Pipe with Thin Steel Cylinder Embedded.—A large and increasing percentage of reinforced concrete pipe is being made by the proc-

ess which has for its prominent characteristic the pre-stressing, into compression, of the concrete core by carefully controlled winding, at a stress in excess of 100,000 lb per sq in., of hard-drawn steel wire having a tensile strength of about 180,000 lb per sq in. The design is such that the concrete does not come into tension at the working water pressure. After the wire is wound on, it is protected by cement mortar thrown on at high velocity and rolled to smoothness.

Small Diameter Pre-Stressed Concrete Cylinder Pipe

During the past 2 years a large producer of reinforced concrete pressure pipe contracted to manufacture approximately 375,000 ft of pre-stressed concrete cylinder pipe in diameters of 20 in. and less. This production represents a large increase over the amount of small diameter pressure pipe manufactured in the two preceding years, 1945 and 1946. During that period only 76,000 ft of small diameter pressure pipe was produced, which represented only 4% of the company's total output of pressure pipe for those years. Makers state that since pre-stressed concrete cylinder pipe is comparable in weight to cast iron pipe of similar diameter, it is possible economically to ship pipe by rail to distant points

Pre-Stressed Reinforced Concrete Noncylinder Pressure Pipe.—Pipe consisting of a concrete core without embedded steel cylinder but pre-stressed by high-tensile wire wound on and the wire protected by an outside coat of mortar was evidently envisaged by European inventors, and some pipe was built in Czechoslovakia and elsewhere previous to 1931, perhaps earlier. In more recent years sewer pipe was so constructed in Illinois. The pipe had pre-stressed longitudinal rods, as well as the wire winding.

For the past few years a corporation formed by a large French and a large American cast iron pipe company has been making large quantities of this pipe without a steel cylinder, pre-stressed both longitudinally and circumferentially. This pipe, in 24-in., 30-in., and 36-in. sizes and in lengths of 22 ft, designed for 150 lb per sq in., is being furnished for the Cuban government in large quantities Joints are made of cast iron rings with rubber gaskets compressed between tapered surfaces. Wall thicknesses are presented in Table 3.

TABLE 3.—Wall Thicknesses of Pre-Stressed Reinforced Concrete Noncylinder Pressure Pipe, in Inches

Diameter	Inner concrete shell	Outer protection	Total
244	1.5	0.75	2.25
30	1.75	0.75	2.50
36	1.9	0.75	2.65

The concrete inner shells are made centrifugally in two piece metal molds. Pre-stressed longitudinal wires 0.218 in. in diameter are attached to the cast iron joint rings which are held to produce a tension of 40,000 ib per sq in. in the wire. The concrete consists of standard Portland cement, carefully graded sand, limestone, and water to make a plastic consistency, thoroughly mixed. Enough concrete to produce one half of the thickness of the inner shell is mechanically placed and the mold spun for 5 min. Then the remaining concrete is placed and the mold is spun for 20 min at a peripheral speed of 3,000 ft per min. When cured the concrete has a

compressive strength of about 13,000 lb per sq in. The concrete is cured first with wet steam at 150° F for 4.5 hours, and then for 2 weeks it is cured in the yard with molds removed.

Wire 0.187 in. in diameter with a strength of from 205,000 to 225,000 and a yield point of from 164,000 lb per sq in. to 185,000 lb per sq in. is then wound on at a stress of 100,000 lb per sq in. An exterior concrete cover coat is placed by a "vibro-buttering" machine. Such pipe, 63 in. in diameter and 5½ in. thick, has been tested in France to a pressure of 795 lb per sq in. without leakage.

Sulfur-Sand Joint Compounds.—The few cases of trouble with sulfur-sand joint compounds, or so-called "lead substitutes," have been the subject of a considerable amount of investigation and discussion during the period covered by this report. In some cases the trouble was confined to the joint itself and in others the cast iron was eaten slightly or to a damaging extent requiring replacement of pipe.

The phenomenon is generally attributed to sulfur bacteria but there is not complete agreement. Extensive research in a few cases seems to have proved that the bacteria were responsible; however, attempts to produce disintegration of the sulfur-sand compound by exposure to a plentiful supply of sulfur bacteria have not been generally successful.

Water engineers and superintendents have the problem of evaluating the risk of disintegration of sulfur-sand compounds against their undoubted advantages in cheapness and in reducing joint seepage by about 75%. Most water authorities are continuing to take the risk involved.

Concrete Pipe Reinforced Spirally.—Reinforced concrete pipe 84 in. in diameter is being made for the City of Montreal, Que., Canada, pre-stressed by spirally wound wires designed to reinforce the pipe both circumferentially and longitudinally.

Quick Operating Valves.—The use of cone valves to reduce the time required in shutting off the flow where a water main is broken has been adopted by Los Angeles and Newark, N. J. At the annual meeting of the American Water Works Association (A.W.W.A.) held in Atlantic City in May, 1948, it was reported that one man can operate the rotary plug valve against full unbalanced water pressure created when a water main is broken. A report from another city indicates that some trouble in operation was experienced.

The need for a reliable valve that will permit prompt shutoff of flow from mains has been shown by general experience. To shut large valves usually requires an hour or more after the break in the main has been reported. It has been suggested that valves designed for extreme reliability and ease of operation, but not necessarily completely tight, might be installed in critical locations in addition to the tight-closing valves already installed.

Masonry Dams

A contribution to the perennial question of uplift pressure was made by Serge Leliavsky Bey,[1] M. ASCE, reporting experiments on the effective uplift

[1] "Experiments on Effective Uplift Area in Gravity Dams," by Serge Leliavsky, in "Uplift Pressure in and Beneath Dams: A Symposium," *Transactions*, ASCE, Vol. 112, 1947, p. 444.

area in gravity type masonry dams, the experiments having been made for the Ministry of Public Works in Cairo, Egypt.

Several investigators have stated that the horizontal water pressure usually considered as applied to the upstream face of a masonry dam actually is totally or partially transferred into the body of the dam by water filtering through the natural pores of the material of which the dam is built.

"Uplift" was defined by Mr. Leliavsky as a "pressure in the pores" or an "interstitial pressure."[2] By the experiments uplift was shown to exist, sufficient to cause failure, by tension, of the material. The uplift force was proved to be a measurable quantity subject to certain laws and capable of analysis.

The average value of the "effective superficial porosity" or ratio of the interstitial area subjected to the hydrostatic pressure, to the total area of the section, was found to be 0.91, with a probable error equal to 0.014. Mr. Leliavsky recommended "* * * 85% as a fairly accurate round figure."[3]

The electric analogy method of analysis was used by Mohamed Ahmed Selim, Assoc. M. ASCE, in experiments on the uplift forces acting on a masonry dam built on porous soil and on the accompanying loss of head.[4] A new experimental technique was developed. The following conclusions were presented:[5]

> "The advantage of an upstream cutoff wall in reducing the uplift force is demonstrated. A downstream cutoff wall tends to increase the uplift force and hence to offset the value of the upstream cutoff wall. The downstream cutoff wall may be necessary, however, to prevent the formation of channels beneath the dam. It is recommended that only a relatively short downstream cutoff wall be used in connection with a relatively deep upstream wall.
>
> "When the thickness of the stratum of porous material under a dam is greater than the base width, the uplift force and moment are essentially the same as when the porous media are of infinite depth."

Drainage of Upstream Part of Masonry Dams.—Another example of drainage provisions, such as have become fairly common in masonry dams since the construction of the dams of the New York City Catskill system beginning in 1910, is the power dam at Stewartsville, Ont., Canada on the Madawaska River. The dam is 1,300 ft long and 200 ft high in maximum section and has a batter of $8\frac{1}{2}$ in. to the vertical foot on the front and a batter of 1 in. to the foot on the back below a vertical face 50 ft high. Vertical steel plates 8 in. wide are used at horizontal joints near the upstream face and drains from these joints are placed just downstream from the plates. Foundation drains with porous concrete collectors are also used.

Earth Dams.—The technique of selecting material, controlling moisture content, and specifying and testing permeability and compaction has become fairly well standardized and is generally applied even to low dams. Frederic P. Stearns, Past-President, ASCE, of Massachusetts made percolation experiments 50 years ago, results of which were used in the design of sand dams of flat

[2] "Experiments on Effective Uplift Area in Gravity Dams," by Serge Leliavsky, in "Uplift Pressure in and Beneath Dams: A Symposium," *Transactions*, ASCE, Vol. 112, 1947, p. 445.
[3] *Ibid.*, p. 486.
[4] "Dams on Porous Media," by Mohamed Ahmed Selim, *ibid.*, p. 488.
[5] *Ibid.*, p. 505.

slopes. Later others made experiments for guidance in design of hydraulic fill dams, but rolled earth dams remained matters of judgment until fairly recently.

The design and construction of earth dams entered prominently into the proceedings of the Second International Conference on Soil Mechanics and Foundation Engineering, held at Rotterdam, The Netherlands, in June, 1948, and the current practice was recognized as sound. According to papers presented there, laboratory tests of soils supplemented by field tests may be used advantageously to supplement judgment in the design and construction of rolled earth dams. With the data obtained from tests of available soils it was reported to be practicable to design the structure so that the materials could be used most economically, and to specify the methods of consolidation of the earth to conform to the requirements of the design. Sheepsfoot rollers and rubber-tired wheels can be used effectively to obtain consolidation of the earth, but there may be a definite limit to the amount of compaction that can be produced with any given set of field equipment, regardless of the number of passes made.

The analysis of the stability of earth slopes by a method of circular arc slides may be used to advantage. Pore water pressure is an important factor in the stability of slopes and should be given careful consideration. Permeability studies with a flow network are helpful in considering the effect of porous materials used in the dam as well as the effect of those found in the foundations. Air entrainment in rolled earth dams may be important.

On large projects the cost of a test fill may be fully justified in making possible a better determination of earth slopes and soil compaction requirements to suit the materials most economically available.

Papers published in the *Proceedings* of this conference are truly international in character, and testify to an increasing world-wide interest in the subject of rolled earth dams.

The committee notes in passing that the modern technique has not resulted in reducing earth dam sections and slopes. On the contrary slopes are generally more conservative than formerly, perhaps influenced in part by the tremendous development in the capacity of earth moving machinery which has kept the price of earthwork more nearly stationary in the face of rising prices than that of any other item entering importantly into the cost of water works.

Spillways

Occasional failures of small dams occur on account of insufficient spillway capacity and only the accidental hit-or-miss nature of torrential rains prevents widespread failures since many dams have spillway capacities only a fraction of those required by recorded floods. Previous reports of this committee have contained comments and data on this matter and there is nothing new to report as far as the committee knows. Practice is so diverse however that the committee thinks the subject should be kept in mind and studied by an appropriate ASCE committee. Some of the diversity is properly accounted for by the degree of damage possible from a failure but it is difficult to justify provisions varying as much as 10 to 1.

Reducing Underground Flow by Impregnation of Asphalt, Also Consolidating Sand by Impregnation with Chemical Solutions

The committee notes, for the benefit of those who have not seen accounts, that methods of reducing underground flow by impregnation of asphalt or of consolidating sand by impregnation with chemical solutions are in use and may bear investigating in case of difficult underseepage or quicksand.

Centrifugal Pumps

A well-known pump company has kindly supplied the committee with information on tendencies in design of centrifugal pumps which have been paraphrased as follows:

Progress continues in casings and impeller vane angles and higher efficiencies are being extended to medium-sized and small-sized units.

The tendency to substitute ball bearings for sleeve bearings continues but experience in the larger sizes has not been too satisfactory and some consulting engineers are going back to actually specifying sleeve bearings for such pumps.

More and more gear teeth type of coupling is being used on larger pumping units.

Additional experience has been gained with mechanical seals and when properly applied they have generally been very satisfactory. For handling gritty liquids a double seal with oil introduced between has been used with success.

Shaft sleeves continue to receive particular attention and the use of 11% to 13% chrome sleeves noted by the committee[6] 2 years ago is continuing with very satisfactory results. For ordinary applications the sleeves are used in a hardness of about 300 Brinell and for severe service they can be hardened to Brinell 500. Stellited sleeves and other surface hardening methods such as "Colmanizing" are also being employed for severe service applications.

The cause of noise and cavitation is still being studied and there is some thought of increasing the specific speed limits now prescribed in the Hydraulic Institute standards. There is substantial opposition to the change. The committee believes that this matter needs further elucidation by pump experts and that the Hydraulic Institute should give further study to the piping approaching and leaving pumps. The present standards prescribe alinements which are often—perhaps generally—violated in the interest of economy and accessibility in pumping station buildings.

One recent trend in water works pumps is the single-suction, vertical pump, liked by engineers because of its compactness and its ability to put piping below the ground floor with direct alinement to underground piping and because the motor is placed on the ground or higher floor where flooding is less likely.

Water Purification and Prevention of Corrosion

Innovations in Mechanical Filtration at the South District Filtration Plant in Chicago, Ill.—This plant has been in full operation since November, 1947, following a period of operation of part of the plant during the 2 previous years.

[6] *Proceedings*, ASCE, April, 1947, p. 517.

It handles water that is highly polluted with industrial waste at times and therefore requires quick changes of chemical dosages. Since the plant was designed to take care of rapidly changing water, no difficulty has been encountered in producing a plant effluent water of high quality which is free of objectionable tastes and odors.

Some difficulty has been experienced in feeding the activated carbon through dry feed machines. Two experimental tanks for handling carbon in the form of a slurry have been constructed and tests are being conducted with the slurry. Although some modifications in the equipment for maintaining a constant strength slurry will have to be made, it now seems likely that this method of feeding carbon will be used in the future.

The possibility of using slurry tanks for carbon storage also is being investigated. Following this investigation there will be a trial shipment of carbon in bulk in hopper cars.

Of special interest to water works officials should be the high rate of filtration used in this plant. The filters are designed for a maximum rate of 5 gal per min per sq ft, using sand 24 in. thick with an effective size of about 0.60 mm and a maximum loss of head of 8 ft—all on the positive side. Since the beginning of full operation in November, 1947, a few of the filters have been set to operate at a 4-gal rate. Careful observation is being made of these high rate units for quality and performance. The quality of the water from such units compares very favorably with that from the units operated at lower rates.

Experience at Chicago indicates that sedimentation must be effective if these high rates are to be successful. This truth is emphasized at Chicago when one basin is out of service for cleaning.

The South District Filtration Plant makes use of sodium silicate for strengthening the coagulation during the winter months. Thus, a filter that would pass coagulated matter quite readily at a 2-gal rate without the silicate during winter months does not pass any coagulated matter when the treatment is used.

It will be recalled that this plant has two-story sedimentation basins which are said to be working even better than anticipated from the small-scale experiments. A part of the sedimentation basin is used as a mixing basin. One third of each basin is provided with continuous sludge-removal equipment. Experience indicates that this equipment should extend over at least one half and preferably over two thirds of the basin.

Provisions were made at this plant for partly softening the water (with a hardness of about 130) to produce a rapidly settling precipitate during periods when microscopic organisms are in abundance. Thus far it has not been necessary to use these provisions. Another reason for not using the softening method is that with it an iron compound is required for coagulation and at present it is not available in sufficient quantities.

Preliminary plans are under way for a filtration plant to serve the central and north water districts of Chicago. The site of the plant has not been decided as yet and when this is done detail planning will go forward as rapidly as possible. A plant to serve these two districts will have a capacity of about 930 mgd—more than twice the capacity of the South District Filtration Plant.

Upward Flow Versus Conventional Type of Pretreatment for Water to Be Filtered.—Water supply engineers, chemists, and operators discussed this matter at considerable length at the 1948 annual meeting of the A.W.W.A. Experience as reported was divided as to the relative advantages of the two methods.

Silica Removal by Iron Shavings.—A successful application of this method has been reported.

Chlorine—How It Kills.—As result of research it is reported that chlorine does not kill by oxidation but by penetration of cells and destruction of enzymes.

Rendering Water Noncorrosive.—The committee regrets to report that no progress appears to have been made in methods of preventing the corrosiveness of water—particularly filtered water—in relation to tuberculation of cast iron and steel water pipe which are not protected by cement lining or by technically controlled bituminous coatings. Consideration is being given, within the ASCE, of a special committee of technicians, with perhaps national application, to make a new attack on this so far impenetrable problem.

"Micro-screening" of Water.—Screening of water by exceedingly fine woven screens of small stainless steel wire is reported to be coming into use in England for sewage and in some cases as a pretreatment for water. The screen is rotated and back-washed continuously. An attempt to exploit the process in the United States has just begun and some consideration is being given to the possibility of using micro-screening as an adjunct to sedimentation.

Proposal for a Limit to Nitrates in Potable Water.—A few years ago a doctor in Illinois traced blueness in a baby to the use of water having nitrates over 100 ppm (computed as nitrogen) in its feeding formula. By inquiry other cases were found where much lesser amounts caused blueness. The blueness is caused by changes in the red corpuscles rendering them incapable of performing their functions. It has been noted only in children under 6 months of age.

Health authorities in the states of Illinois, Iowa, and Ohio are reported to be considering placing a limit on nitrates in potable water. The Town of Hempstead, Long Island, N. Y., has recently refused permission for the construction of a proposed well partly because four adjacent wells have nitrates (N) varying from 8 ppm to 13 ppm and averaging about 10 ppm. Another objection was that the proposed well would have had the same depth as that of the existing four wells—about 60 ft. It was ruled that the additional well should be deeper to give correspondingly greater protection against pollution and with the expectation of less nitrates so as to bring down the average from the station.

The committee knows that well water which had nitrates something over 20 ppm has been used for years without known trouble. It suggests that the Sanitary Engineering Division should join in country-wide action, and if necessary take the initiative in such action as would lead to the establishment of a proper limit to nitrates. There is a present tendency to fix the limit as 10 ppm of nitrates, computed as nitrogen (N). This limit may be unnecessarily low and might cause unnecessary expense in the abandonment of wells constructed at great cost.

Chromium in Water Supplies.—Danger of chromium in objectionable quantities in ground-water supplies has been found to exist in one locality on Long Island caused by the wastes of an industrial plant.

Cathodic Protection for Steel Tanks.—A scholarly set of tests has confirmed the theoretical correctness of the principle of cathodic protection for tanks—that a current of electricity flowing from anodes suspended in the water of the tank to the steel shell will inhibit corrosion provided an adequate installation is made. The installation in which attachments to the exterior of the tank replaced interior anodes was not effective in the experiments. Carefully planned and checked evidence as to the effectiveness of the application of anodes installed in full-sized tanks—that is, proof that the applications are adequate—does not seem yet to have been assembled.

Cathodic protection for exterior of pipe lines is treated elsewhere, following the heading, "Pipe Materials."

Valuation of Public Utilities

The 1944 and 1946 reports of this committee referred to the United States Supreme Court decision in the Hope Natural Gas Case, which provided more power and freedom to regulatory bodies in establishing rates. In that case the court, in overruling a decision of the United States Circuit Court of Appeals, indicated that the "end result" obtained by the Federal Power Commission (FPC) was not confiscatory, although the FPC ignored evidence of present value of the property at current price levels, inasmuch as the rates established by the FPC permitted "* * * The Company to operate successfully, to maintain its financial integrity, to attract capital, and to compensate its investors for the risks assumed." Although this decision did not directly bar consideration of fair value as measured by cost of reproduction as a factor in rate making, it did strengthen the position of those advocating rates based on costs at original price levels rather than on fair value at current price levels. There has been no change in this situation since this decision and rate regulatory bodies in general recently have given little consideration to reproduction cost, except in those states whose laws prescribe that it must be considered as one of the factors in determining whether or not rates are reasonable.

Since the 1946 report was published, the results of the activities of the A.W.W.A. Committee on Survival and Retirement Experience with Water Supply Facilities have become available in a bound volume published by A.W.W.A.

Water Rates

The postwar period, and particularly the period covered by this report, has experienced a rapid increase in the cost of producing, distributing, and selling water. In the case of private systems, there also has been a substantial increase in the proportion of revenue required for taxes. Although, in general, gross revenues have increased, the net income of many water systems under prewar rates has declined to an alarming degree. This reduction has resulted in curtailing improvements and additions necessary to provide adequate service. Louis R. Howson, M. ASCE, has pointed out the extent of the accumulated lag in water works construction during the past 8-year period and the fact that

rates must be increased to maintain a satisfactory standard of service and to provide for growth.

Private water companies have been hesitant in becoming involved in lengthy valuation proceedings to obtain increased rates, but many of them have succeeded in obtaining temporary increases without costly litigation, based on proofs of increased operating costs and the need for higher return to permit financing of improvements. Municipal water departments generally have been more hesitant in attempting to increase rates, largely because of political considerations. However, many of them have increased rates or are considering this and other methods of obtaining additional funds. Others should give thorough consideration to their ability to provide proper service under existing rates.

The problem of financing sewerage facilities is similar to that of financing water works and methods of charging for the service often are related to charges for water service. The last 2 years has shown an increased interest in financing sewerage facilities by charging individual property owners directly for the service. A number of large cities including Philadelphia, Louisville, and Akron, Ohio, have established rates to provide income for financing and operating sewerage facilities. Flat rates, fixture rates, and rates based on quantity of water used have been employed extensively as bases of charges for sewerage service. Modifications of these methods will be developed as the need for them arises. Interest in this problem has been evidenced by the ASCE by its establishment in 1948, under the Sanitary Engineering Division, of the "Committee on Fundamental Considerations in Rates and Rate Structures for Water and Sewage Works," under the chairmanship of S. A. Greeley, M. ASCE. This committee comprises members of the A.W.W.A. and of the Federation of Sewage Works Associations who are also members of the ASCE and who have given much study to this subject. Also cooperating with this committee are representatives of the American Bar Association and the Municipal Finance Officers Association.

Radio Telephones.—A marked increase has occurred in the number of water supplies which employ radio telephones for communicating to and between motor vehicles on the road. There are two classes of such service—by direct radio communication and indirectly through telephone centrals. It has been suggested that water works which should have this service should apply promptly because the wave lengths are limited and less important businesses might come to take up wave lengths which water supplies should have.

Frequency modulation provisions are understood to greatly increase the number of different businesses that can be accommodated.

Respectfully submitted,

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Committee of the Sanitary Engineering Division on Water Supply Engineering

January 20, 1949

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

CITIES CANNOT LIVE WITHOUT TRUCKS

Discussion

BY HOY STEVENS

HOY STEVENS[13].—The discussions by Messrs. Lewis, Miller, and Nees indicate a growing appreciation of the problems of commercial transportation in urban communities. Because of the significance of commercial vehicles in industrial cities, it will require cooperative effort of architects, builders, city planners, and truck operators in each community to develop realistic future plans.

Mr. Lewis further stresses the need for formulas by which off-street loading areas and truck berths for commercial buildings can be determined. As he states, a building's use is a predominant factor in the number of truck berths required. More study and data are needed on this subject. Also, in calculating off-street loading areas, consideration must be given to the size of the vehicle that will use these docks. Currently there is a trend toward 16-ft bodies on city freight delivery trucks. Generally these 16-ft bodies are mounted on cab-forward types of motor trucks, but the over-all length is increased slightly.

A quick rule for width of manipulation areas in front of a dock is that the area width from the dock to the limiting boundary shall be not less than twice the over-all length of the vehicle. A greater width makes for easier manipulation and backing into truck berths, and is recommended. The same rule holds true for the longer combinations of tractor semitrailers.

In regard to his recommended supplementary type of regulations, Mr. Lewis' suggestions would not bar tractor semitrailer combinations from making truck load deliveries. With this view the writer is in accord. In general, in congested areas most carriers find it advantageous to make less-than-truck-load "pickups and deliveries" with city trucks, using only road semitrailers for truck-load movements. The economies of these practices are readily apparent in less manual handling of the freight.

NOTE.—This paper by Hoy Stevens was published in December, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1949, by Harold M. Lewis; and May, 1949, by John A. Miller, and L. A. Nees.

[13] Chf., Equipment and Operations Section, Am. Trucking Assns., Inc., Washington, D. C.

Mr. Miller restates an error that appears frequently in many traffic surveys, when he states that trucks are found partly empty while running in the streets. This is inevitably the case when the amount of loading in the trucks is checked at outlying survey stations. The only time a city delivery truck making deliveries can be full is just when it leaves the carrier's terminal. From then on it is becoming more and more empty as it makes deliveries. This must be the case if the truck makes deliveries. The contrary is true with a pickup truck collecting outbound shipments. In this case the truck is full only between its last pickup stop and the carrier's terminal. This partial empty phenomenon has been recorded in many traffic surveys, and the incorrect analysis of the causes for the phenomenon have resulted in very erroneous conclusions about city truck operation.

Mr. Nees' suggestion regarding a code for off-street loading facilities indicates a reasonable approach to the problem, both as regards new construction and old buildings.

Mr. Nees' objections to the writer's suggestion regarding the size of a loading dock further point out the interrelation between intrabuilding transportation facilities, such as elevators or industrial dock trucks, and dock areas. Since the dock area (even following the broad suggestions in the paper) will be less than the area required for the motor truck manipulation area, there are advantages in dock arrangements (especially in buildings) which will enable the fast loading and unloading of vehicles. The faster that motor vehicles can enter and leave a dock area, including either loading or unloading time, the smaller can be the truck manipulation area and the fewer can be the truck berths required for a given tonnage. For this reason, there are real advantages to a building owner in having his dock and his material handling equipment as efficient as possible for both the loading and unloading of commercial vehicles, and for the moving of the freight within the building.

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DISCUSSIONS

NOMOGRAPHIC ANALYSIS OF RECTANGULAR SECTIONS OF REINFORCED CONCRETE

Discussion

BY W. P. LINTON

W. P. LINTON,[16] M. ASCE.—The reception of this paper, both in the published discussions and in personal correspondence, was gratifying to the writer. The purpose of the paper was to reduce the labor involved in the design and checking of reinforced concrete sections subject to bending and direct stress, while adhering to the accepted theory of reinforced concrete design.

Mr. Barron presents an example that tends to show a paradox in the straight-line theory. The writer considered the straight-line theory to be the accepted theory and the nomograms were made accordingly. Mr. Barron also states (truthfully) that:

> "Since the compression bar in Fig. 4 replaces an equal area of concrete, p' should be multiplied by $(n - 1)$ instead of by n in the derivation."

Mr. Hirschthal writes:

> "It is all the more regrettable, therefore, that he did not take into consideration the fact that both the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete[8] and the American Concrete Institute in the latest design specifications[9] have given consideration to the fact that the modular ratio in the compression side varies from that of the tension side of a concrete member reinforced for compression as well as tension. The regulation adopted by both these bodies is based on the assumption that the value of n (the modular ratio) for compressive reinforcement is double the value used for the tensile reinforcement. This assumption was adopted because of the low stresses to which compressive reinforcement was subjected under the old assumption of a uniform n on both sides of the neutral axis of the section, and was the result of test ob-

NOTE.—This paper by W. P. Linton was published in January, 1949, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1949, by Maurice Barron, and M. Hirschthal; and June, 1949, by Winfield A. McCracken, King Burghardt, and Robert V. Hauer.

[16] Senior Highway Bridge Engr. (*Retired*), U. S. Public Roads, Philadelphia, Pa.

[8] "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," *Proceedings*, ASCE, June, 1940, Pt. 2, p. 45.

[9] *Journal*, A.C.I., June, 1945, p. 582.

servations[10] made on large size columns at the University of Illinois, at Urbana."

What Mr. Hirschthal states is true, but not so regrettable as one might, at first, think, because it would make no change in Figs. 1 and 2 except in the label of the $(p'\ n)$-scale. Those nomograms can be used as they are by simply giving any value desired to n. The value of n, to be used in Fig. 2, need not be the same as the value used in Fig. 3. Mr. Hirschthal's suggestion would make a change in Fig. 1. As stated, Figs. 2 and 3 can be used as they are with the understanding that n may have different values in the two nomograms.

Mr. Burghardt proposes two additional nomograms, Figs. 6 and 7. The writer believes that Fig. 6, the nomogram for finding k in a given section, is a very useful and desirable addition. Although the stresses in a given section can be checked by Figs. 2 and 3 alone, as is done in the paper, the direct method, by using Fig. 6, now seems to the writer to be more desirable. Fig. 7, the nomogram for equal steel in tension and compression, will be useful (as stated by Mr. Burghardt) in some cases.

Mr. Hauer calls attention to other charts,[15] which in his opinion are superior in several respects. The writer cannot comment on them because he has not seen them. Mr. Hauer also feels that:

> "* * * it is regrettable that the author did not extend the nomograms far enough to cover the range of small eccentricities, which, at least in building design, is the case most often encountered."

That is the case in which there is no tension on the concrete, and the solution of that problem is not as difficult by other methods, as it is where there is tension. However, it would be desirable to have that case covered by nomograms.

In conclusion, the writer believes that nomograms, in general, are very useful and can, and should, be used much more extensively than they have been. In some cases it is more difficult to construct a nomogram than a diagram, but when constructed, the nomogram is more easily used and more variables can be provided for.

After considering all the discussions, the writer is of the opinion that Figs. 2 and 3 (with the n on the $(p'\ n)$-scale in Fig. 2 changed to n') together with Mr. Burghardt's Figs. 6 and 7, make very useful and convenient tools for the purpose for which they were intended.

The writer wishes to thank all those who have taken an interest in the paper.

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[10] *Bulletin No. 40*, Univ. of Illinois, Urbana, Ill., June 5, 1934, p. 86.

[15] "Der Eisenbetonbau," by E. Moersch, Konrad Wittwer, Stuttgart, Germany, 5th Ed., 1920–1922

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DISCUSSIONS

THE LOGICAL SYSTEM OF FREIGHT RATES

Discussion

BY CHARLES L. HALL

CHARLES L. HALL,[15] M. ASCE.—That some transportation agencies have been set up, and are maintained, for noncommercial reasons (as suggested by Mr. Courland) has undoubtedly been true of many agencies in the past, and may still be true of air lines. It should not be true now of any other carrier within the continental limits of the United States. The problem of rate making is only slightly affected by the existence of navigable rivers equipped with multiple purpose dams. The amount of commerce carried by the Tennessee, Muskingum, and Savannah rivers in not likely to cause any serious fear in the minds of competing carriers.

In his brief discussion, Mr. Meyer recommends a course of action which it is the object of the writer to prevent at all costs. If a commodity is not produced or shipped in some given area without a freight rate concession, it can be produced elsewhere. It is a pure myth (as Mr. Faison points out) that a freight rate concession can be given to any item, without throwing a burden on some other item, or (as in the case of upper Mississippi barge traffic) on the general taxpayer. Let the regional development be a little slow. The United States has plenty of time.

The writer particularly welcomes Mr. Markowitz's criticisms. They present a direct attack to which a reasoned reply can be made. The value of the commodity, to which he states that insufficient attention is given, is allowed for at the time of classification of freight, which is the first step in the writer's suggested process. His expression, "service on which the ability to pay in freight rate making is based," presumably refers to the type of carrier used by the shipper. Differences between types of transportation are certainly very carefully allowed for in the plan presented.

The basic paper is strongly criticized for presenting a scheme that would prevent competition. It is not clear whether the competition prevented is

NOTE.—This paper by Charles L. Hall was published in November, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1949, by Raphael H. Courland; February, 1949, by Otto Meyer; April, 1949, by Alexander Markowitz; and June, 1949, by Haywood R. Faison, and Malcolm Elliott.

[15] Col., Corps of Engrs., U. S. Army (*Retired*), Washington, D. C.

between carriers of the same type, between carriers of different types, or between communities. Carriers of the same type do not compete now, in price. They all quote identical rates for the same cargo between the same cities and all of them try to offer better service. Under the new plan, they can continue to do so. Carriers of different types—notably the railroad and the river boat—have competed for many years; but this competition has almost invariably resulted in increasing freight rates to or from destinations having access to only one type of carrier. It is this arbitrary shifting of the burden which the writer's plan attempts to prohibit. Competition between communities is created by one or more carriers granting a favored position to some city, increasing their rates to other cities in compensation. No student of economic history truly believes that equalization of rates to competing seaports has been to the nation's advantage. For example, the gift, to Baltimore, Md., of its natural freight rate advantage from the Ohio River valley, instead of an insistence on the present so-called equalized rates to New York, N. Y., would have caused an advantageous decentralization in the United States. A port with freight rates against it would have to develop some advantage in service to keep its commerce, or it would have to specialize in supplying its natural market. It is admittedly intended to destroy competition based on penalizing communities provided with only one carrier, or type of carrier, as well as to injure the ports that have insufficient "pull" to avoid the same kind of involuntary servitude to places of more influence.

The last paragraph of Mr. Markowitz's discussion is certainly a vigorous summing up. The writer has spent 30 years arguing about rates with all the traffic men whom he has ever met. He is thoroughly accustomed to being disagreed with; but no traffic man has ever before painted the dark picture of coming evil portrayed in this discussion. Much can be said for free competition between all carriers, but, unfortunately, that method was found not to work. When competition does not work, some kind of public control is inevitable.

The writer desires to thank Mr. Faison, whose discussion greatly increases the value of the paper. In particular, his discussion of the fallacy of the so-called "added traffic formula" is a welcome supplement to the arguments presented by the writer. During the discussion of this paper the writer has thought a great deal about the advisability of increasing ton-mileage charges on routes of small traffic density. It is believed that a rough rule of thumb would be to increase the actual mileage, for computing purposes, in counties having a population density more than 10% below the national average. Population and traffic go quite well together. Under the logical system of rates, the only effect of the milling-in-transit privilege would be to eliminate one loading and one unloading charge. Except in the case of export wheat, it would seem unimportant to the nation whether the privilege is granted or not. It would probably be to the national advantage to avoid concentration of mills at seaports.

The writer agrees with Mr. Faison that there is no fair method of applying a transportation tax to any carrier unless rates on all carriers are based on costs of service. Certainly the immediate effect of a tonnage tax on either motor

trucks or river boats, right now, would be merely an increase in competing rail rates. Needless to say, no Congress would adopt such a novel form of taxation for the exclusive benefit of rail carriers. The tonnage tax might arrive after the logical system is under way, but the immediate step is to prevent any such economic absurdities as the extension of government-owned inland water lines operating at a loss, or the payment of a marine subsidy to coastwise ocean traffic.

As to Mr. Faison's warning about the dangers of a too precipitate adoption of this proposed system, it may be stated that undue haste seems the least probable of all contingencies. It is much more likely that the plan will be dismissed as an innocent theoretical exercise. Its only hope of adoption is as a consequence of a regional revolt like the flare-up on southern freight rates in 1947.

In reply to Colonel Elliott, the writer admits that the logical system is based on the theory that all freight rates should be remunerative, and that the disappearance of some carriers and some communities is a probable consequence of the plan's adoption. Any reform in rate making is certain to "step on somebody's toes." If California oranges tend to cost too much for New Englanders to eat, they will have to learn to be satisfied with Florida fruit. There is no particular reason why people in Utah should pay more for their lemons in order to extend an artificial market into the northeast at noncompensatory rates.

The writer is thankful to his critics and is sorry that there were not more of them. He is particularly sorry that there was no discussion of his plan to break all freight rates into separate terminal and actual transportation charges. This seems to him to be the most important single feature of the proposed system of rates.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE MUNICIPAL ASPHALT PLANT, BOROUGH OF MANHATTAN

Discussion

BY L. CSANYI

L. CSANYI,[13] M. ASCE.—Messrs. Oglesby, Henderson, and Holmes have introduced into their discussion an age-old controversy which will range on for many years to come—namely, public methods versus private methods of financing and accounting. Although this cannot be settled here, it is only fair since one side has been portrayed to present the other also.

In public financing, regardless of the method, the primary object must always be: Does the taxpayer receive full value? To assure that he does, therefore, each project must be considered upon its own merits and not by generalities. This is particularly true of operating plants, and holds alike for public or private enterprise. Furthermore, those services rendered which do not reduce readily to a monetary basis must be evaluated and given proper proportion in the final analysis. Thus, general formulas do not readily apply without extensive and proper adjustment.

As a case in point, it would be difficult if not impossible to assign a value for i (risk), in Eq. 1, for a plant whose full output is assured as long as it operates. Also, if this had been a private enterprise, the value would have the same significance, with $i = 0$, $R = 0$. It is apparent, then, that the method used gives a reasonable value for R.

Another omission cited is interest on land. The conditions that permit such an omission are as follows: The land, originally a swamp, was purchased for a very small sum many years ago. Private enterprise was glad to be rid of it; yet when the City of New York (N. Y.) converted it to a convenient dump for the disposal of excavated materials, private enterprise as well as the the city used it gladly to save money. In 1913 the city reclaimed the filled area for the site of the old asphalt plant; 27 years later the construction of the East River Drive enhanced the land values in this area to a point where this site was valued at $3,000,000. The reasons why the land was retained as the

NOTE.—This paper by L. Csanyi was published in November, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1949, by Clarkson H. Oglesby, Clark D. Henderson, and Oscar C. Holmes, Jr., and J. H. Creed.

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site of the new plant are adequately indicated in the paper. Under these conditions interest could not properly be charged on original cost, actual cost (including the large savings made), or present value. A taxpayer required to pay this interest charge in the cost of materials used to pave an emergency water pipe repair to his house would have a legitimate complaint.

Taxes fall into a similar position as interest under these conditions. Considering land alone, there is substantially nothing to tax. The discussers hold that private enterprise would have to pay in the same situation, and that this sum has been removed from the tax rolls. This argument is open to question. Another site equally expensive but less efficient would still reduce tax rolls and cost more. Why build the plant at all? To carry this reasoning to the ridiculous, why should the city build anything? Why not turn back the streets, they are valuable and would greatly increase the tax rolls? The answer is to be found in whether the taxpayer gets a fair return.

The problem of taxes on the improvement poses similar questions, dependent on specific conditions. This plant operates when the management of private enterprise deems operation uneconomical or inexpedient. Emergencies exist during rainy days and inclement winter weather. These emergencies must be met and damage must be repaired if public life and property are to be protected. It is true that the production costs rise when the plant produces from only 30 tons to 50 tons on these days. Estimated cost of production is $25 a ton; yet this rate is averaged in on an annual basis and the taxpayer is charged at the nominal rate. If this plant were not in existence, and private enterprise were induced to meet the need, the cost per ton would be nearer $50. Since the plant operates about 100 days per yr under these conditions, a saving of 40 tons per day for 100 days at $25 a ton would amount to $100,000 per yr. To the taxpayers paying the specific bill for this material, it will amount to $180,000 per yr. Taxes at an average of $3.00 per hundred for $1,200,000 equal $36,000 on improvement, or $126,000 on land at present value plus improvement. Taking the difference equal to $26,000 in the latter, this is not too high a cost for services rendered in protecting life, limb, and property.

Depreciation is a major dispute not only in public enterprise but also in private enterprise. Again, examine the conditions. The vast changes referred to in the discussion do not apply to that manufacturing where only refinements have been developed. Furthermore, the plant as designed anticipates future modifications to be readily incorporated. Thus, sudden obsolescence is not expected. Therefore, 20 years of life for equipment seems reasonable, particularly when the old plant operated 27 years without major replacement. As to equipment cost, this included such items as bins, electrical controls and wiring, supporting structure and foundations for machinery, tanks, piping, and other items which have a life expectancy in excess of 20 years. The discussion calculated a depreciation reserve of $250,000 in 20 years at 2%. This sum is more than enough to meet the replacement cost of the operating machinery. Furthermore, the large annual costs included under maintenance, and preventive maintenance under labor; provide leeway for long life of both equipment and structures. (Payroll, purchasing, supervision, etc., have been covered under supervision in labor cost breakdown.)

This is the other side of the argument. The reader should appraise the whole as a taxpayer as well as an economist.

Mr. Creed has indicated an interest in the reasons behind the selection of electrical precipitation for last-stage dust control. Both methods mentioned and many others were studied before final decision. In all these methods the purity of the stack exhaust proved to be below the desired result. Other objectionable problems were introduced also—namely, collection and disposal of sludge produced and loss of dust. Since the paper was prepared, this installation has been placed in operation. The system, the first of its kind used for this type of work, functions excellently under a wide range of stack temperatures, steam content of flue gases, and gas volumes. The combined system removes 99.7% of dust, makes the stack discharge purer than the surrounding atmosphere, and frees the plant from reliance on favorable winds.

Although the original manuscript (on file in the Engineering Societies' Library[13a]) contains a breakdown of personnel, a brief review here may serve a worthy purpose:

Department	Function	Number of men	
Supervision and office	Engineer in charge	1	
	Foreman	1	
	Clerk	1	3
Materials handling	Craneman	1	
	Barge trimmers	2	
	Conveyor operators	2	5
Manufacturing paving materials	Conveyor operator	1	
	Drier man	1	
	Mixer man	1	3
Maintenance and repair (regular and preventive; also asphalt and dust handling)	Master electrician	1	
	Electrician	1	
	Electrician's helper	1	
	Master mechanic	1	
	Mechanic	1	
	Mechanic's helper	1	
	Oilers	2	8
General	Laborers		2
Total			21

From this breakdown, it will be noted that only thirteen men are required to operate the plant. The other eight are used for extensive preventive maintenance and repair of equipment and structures. The cost per ton is slight for the return in long efficient life required of a municipally operated plant.

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DISCUSSIONS

DEVELOPMENT OF CBR FLEXIBLE PAVEMENT DESIGN METHOD FOR AIRFIELDS A SYMPOSIUM

Discussion

BY S. M. FERGUS, RAUL VALLE-RODAS, TOM W. STALLWORTH, ROBERT HORONJEFF, JOHN J. O'MARA, AND D. P. KRYNINE

S. M. FERGUS.[54]—Messrs. Middlebrooks and Bertram in their paper state that one of the methods used in extrapolating the California bearing ratio (CBR) design curves to heavier wheel loads was the elastic theory. In their extrapolation it was the intent that sufficient thickness of base and wearing course be provided to prevent subgrade shear. The thicknesses obtained in this manner, however, were empirically decreased for the higher CBR values on the basis of the experience of the consultants. Mr. Foster presents data relative to the behavior of several pavements under traffic tests and compares the results by appropriate wheel load groups. It can be shown that the CBR design curves currently appearing in the *Engineering Manual for War Department Construction*[33] follow the elastic theory very closely and that, therefore, they may be expressed as a single curve which is applicable to any wheel load. Also, the traffic test data presented in Mr. Foster's paper may be plotted to the same scale and compared with this single design curve.

Fig. 38 represents any two independent uniform circular loads placed on the surface of a semi-infinite, elastic body. Point P′ corresponds to point P just as a point on a photographic enlargement corresponds to the same point on the original; that is, the depths and offset distances are the same in terms of the respective radii. Assuming that both loads have the same surface contact pressure, it is well known from elastic theory that the value of any particular stress (vertical, horizontal, principal, or shear) at point P′ is equal to the value of the same stress at point P. If a comparison is made of a series of such uniform

NOTE.—This Symposium was published in January, 1949, *Proceedings*. Dicussion on this Symposium has appeared in *Proceedings*, as follows: May, 1949, by Miles S. Kersten, and F. C. Tyrrell and L. A. Palmer; and June, 1949, by Jacob Feld.

[54] Engr., Flexible Pavement Branch, Soils Div., Waterways Experiment Station, Corps of Engrs., Vicksburg, Miss.

[33] "Airfield Pavement Design: Flexible Pavements," *Engineering Manual for War Department Construction*, Office of Chf. of Engrs., U. S. Army, Washington, D. C., July 1946, pt. XII, p. 11, Fig. 2.

circular loads which have the same contact pressure but are of increasing total magnitude, it can be noted that the value of any stress can be maintained constant at corresponding pionts such as P and P′ by maintaining the ratio of the depth to the radius of the loaded area at a constant value. Moreover, since P and P′ are any corresponding points, they can as well be taken at the location of the most critical stress at the depth under consideration.

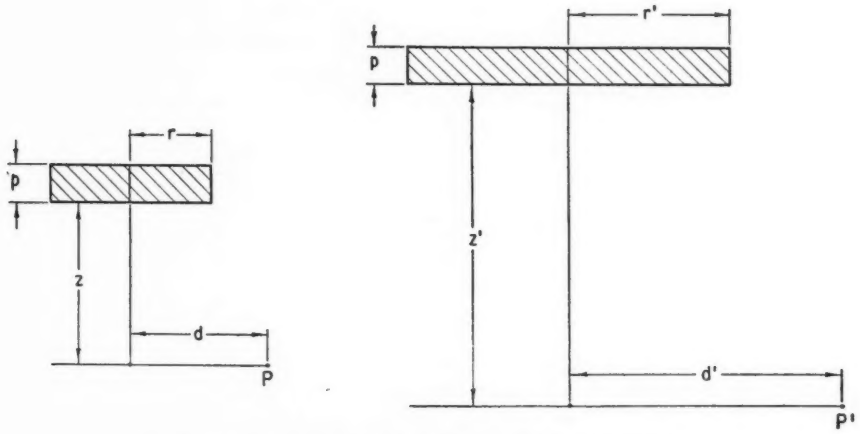

FIG. 38.—CORRESPONDING POINTS BENEATH UNIFORM CIRCULAR LOADS

The original CBR curves were extrapolated by the consultants in different ways. Messrs. Middlebrooks and Bertram made their extrapolations on the basis of the elastic theory, increasing the thicknesses for the different wheel loads sufficiently that the maximum shear stress for a given CBR value was held at a constant value. This result can be accomplished readily, by the photographic enlargement concept described in the preceding paragraph, by maintaining the ratio of depth to radius of loaded area constant for any given CBR value. As described in the latter part of the paper by Messrs. Middlebrooks and Bertram Mr. Porter's curves were based on an allowable deformation for the various loads, whereas those of A. Casagrande, M. ASCE, were based on the relative sizes of the loaded areas. However, since the curves obtained by the three different methods were in substantial agreement, it appears that the same relation between the thicknesses and the dimensions of the loaded areas is inherent in each and, therefore, a constant ration of depth to radius of loaded area can be assumed for any given CBR. This relation is expressed by the equation:

$$z = a\,r = a\sqrt{\frac{P}{\pi\,p}} = \frac{a}{\sqrt{\pi\,p}}\sqrt{P} = K\sqrt{P} \quad \ldots\ldots\ldots\ldots (1)$$

in which z is the combined thickness of wearing course and base, in inches; r is the radius, in inches, of the loaded area treated as a circle; a is the ratio of the depth z to the radius r, a constant which depends on the CBR of the subgrade; p is the surface contact pressure, in pounds per square inch; P is the total wheel

load, in pounds (equal to $\pi r^2 p$); and K is a constant (equal to $a/\sqrt{\pi}\ p$) if p and CBR are constant.

The thicknesses for wheel loads of from 5,000 lb to 150,000 lb required by the CBR design curves published in the *Engineering Manual for War Department Construction*[33] are given in Table 12. The values shown for the 200,000-lb wheel load are those derived in 1948 from Stockton, Calif., test section No. 2. In each case the value for K has been computed from the relation:

$$K = \frac{z}{\sqrt{P}} \qquad (2)$$

Reading from left to right in Table 12, it can be noted that, for any particular CBR, the values for K tend to be nearly equal and that there is no particular trend with the load, the values for the smaller loads being in some cases greater than those for the heavier loads. However, those for the 40,000-lb wheel load

TABLE 12.—PAVEMENT THICKNESS REQUIRED FOR GIVEN WHEEL LOADS (1 KIP = 1 "KILO"-POUND = 1,000 LB)

California bearing ratio	5 Kips		15 Kips		40 Kips		70 Kips		150 Kips		200 Kips		Average
	z	K	z	K	z	K	z	K	z	K	z	K	K
3	14.0	0.198	25.0	0.204	37.5	0.188	49.6	0.187	75.0	0.194	89.5	0.200	0.195
4	12.2	0.173	21.4	0.175	31.7	0.159	40.7	0.154	64.0	0.165	75.5	0.169	0.166
5	11.0	0.156	18.8	0.154	27.7	0.139	36.0	0.136	56.5	0.146	66.5	0.149	0.147
6	9.9	0.140	16.9	0.138	24.9	0.125	32.2	0.122	50.7	0.131	60.0	0.134	0.132
7	9.2	0.130	15.4	0.126	22.6	0.113	29.3	0.111	46.0	0.119	54.5	0.122	0.120
8	8.5	0.120	14.1	0.115	20.7	0.104	26.9	0.102	42.5	0.110	50.5	0.113	0.111
9	8.0	0.113	13.0	1.016	19.2	0.096	25.0	0.094	39.8	0.103	47.0	0.105	0.103
10	7.5	0.106	12.1	0.099	17.9	0.090	23.3	0.088	37.3	0.096	44.4	0.099	0.096
15	5.4	0.076	8.9	0.073	13.4	0.067	17.9	0.068	29.0	0.075	35.0	0.078	0.073
20	....	....	6.7	0.055	10.6	0.053	14.7	0.056	25.0	0.065	28.8	0.064	0.059
30	....	....	....	....	6.9	0.035	10.5	0.040	19.5	0.050	20.9	0.047	0.043
40	....	....	....	....	4.4	0.022	7.6	0.029	16.5	0.043	16.3	0.036	0.033
50	....	....	....	....	....	....	5.6	0.021	14.0	0.036	13.3	0.030	0.029
60	....	....	....	....	....	....	....	....	12.5	0.032	11.2	0.025	0.029
70	....	....	....	....	....	....	....	....	....	....	9.5	0.021	0.021
80	....	....	....	....	....	....	....	....	....	....	8.3	0.019	0.019
90	....	....	....	....	....	....	....	....	....	....	7.4	0.017	0.017
100	....	....	....	....	....	....	....	....	....	....	6.6	0.015	0.015

and the 70,000-lb wheel load are somewhat less than the others. To determine how much change in the design thicknesses would be involved by assuming that K is constant for a particular CBR, thicknesses for each of the wheel loads were computed from the average values for K shown in the right column of Table 12. These thicknesses are plotted and compared with the original curves in Fig. 39. A comparison of the two sets of curves shows that there is good agreement for the 5,000-lb curve and the 15,000-lb curve and for that part of the 150,000-lb curve with a CBR less than about 15. The agreement for the 40,000-lb curve and the 70,000-lb curve is not as good, the present design curves showing somewhat less thickness than the computed values. As stated by Mr. Foster, however, the behavior data indicate that the thicknesses for these two wheel loads may have to be increased. The computed thicknesses for the 150,000-lb curve are less than those required by the present design curve above a CBR of about 15. The thicknesses developed at Stockton for the 200,000-lb curve for these CBR values indicate that the lesser thicknesses for the 150,000-

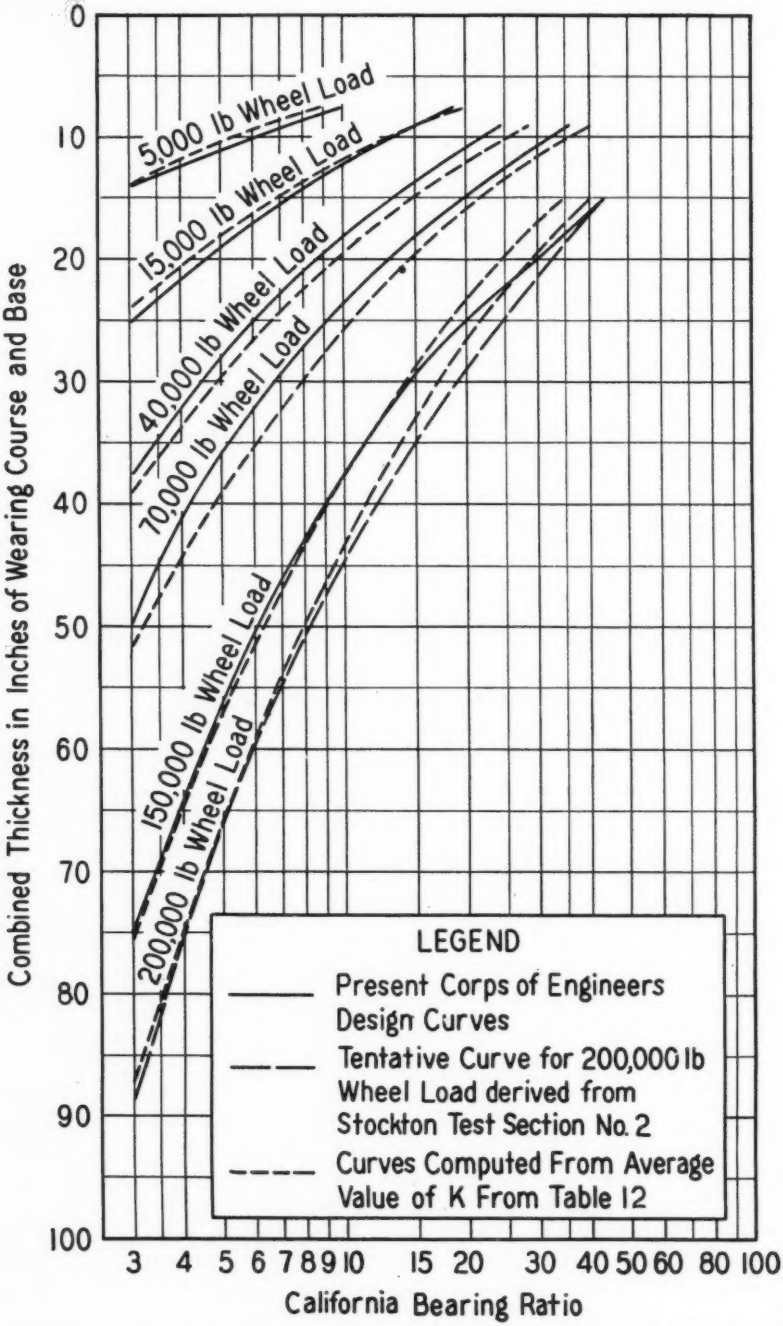

FIG. 39.—COMPARISON OF PRESENT DESIGN CURVES WITH CURVES COMPUTED FROM AVERAGE *K*-VALUE

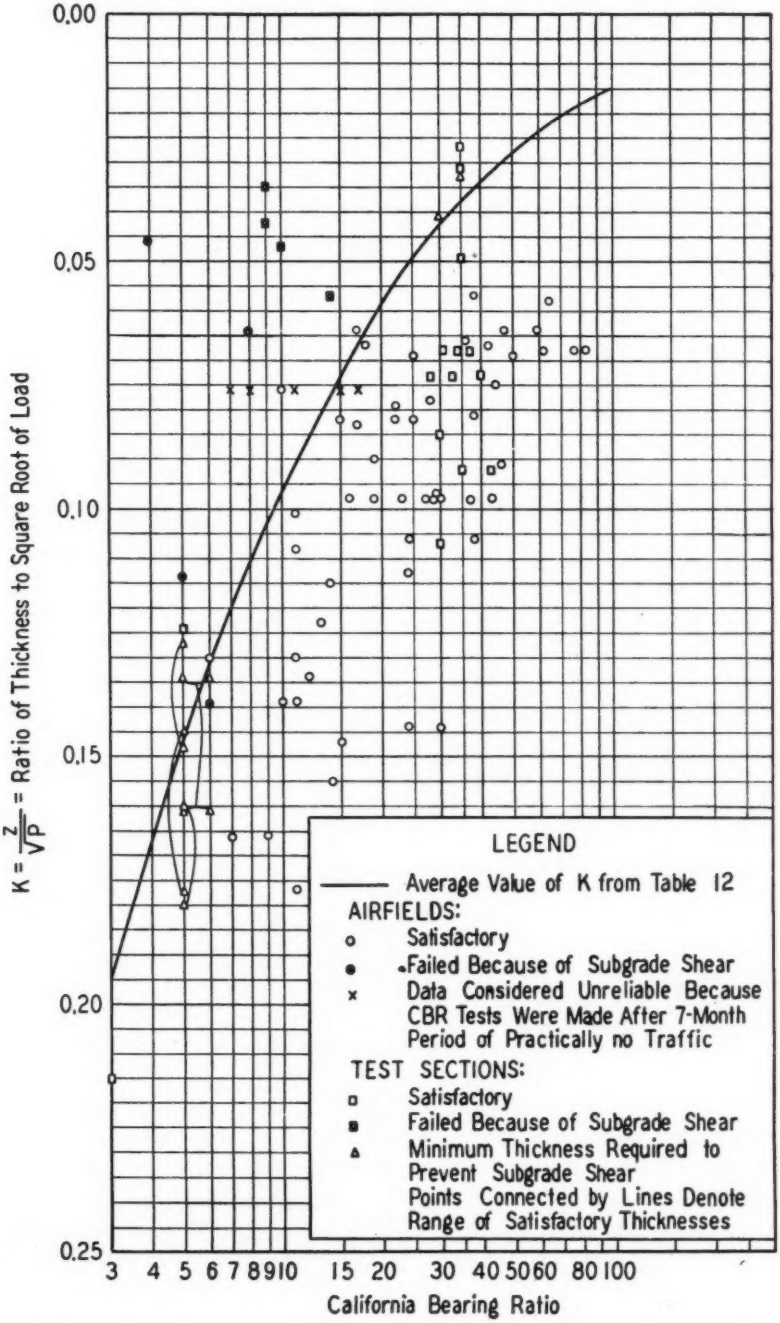
0.00
0.05
0.10
0.15
0.20
0.25
$K = \frac{Z}{\sqrt{P}}$ = Ratio of Thickness to Square Root of Load
3 4 5 6 7 8 9 10 15 20 30 40 50 60 80 100
California Bearing Ratio
LEGEND
Average Value of K from Table 12
AIRFIELDS:
Satisfactory
Failed Because of Subgrade Shear
Data Considered Unreliable Because CBR Tests Were Made After 7-Month Period of Practically no Traffic
TEST SECTIONS:
Satisfactory
Failed Because of Subgrade Shear
Minimum Thickness Required to Prevent Subgrade Shear
Points Connected by Lines Denote Range of Satisfactory Thicknesses

FIG. 40.—CORRELATION OF AVERAGE DESIGN CURVE WITH AIRFIELD EVALUATION DATA

lb load may be more reasonable. In view of the generally good agreement between the two sets of curves, the assumption that K can be taken as a constant for a particular CBR seems reasonable.

The use of the single factor K for the expression of the thicknesses for any wheel load makes it possible to plot all the available traffic test data to the same scale. Using values for z and P as shown in Table 9, the thicknesses at which the several pavements failed or were satisfactory were converted to K-values using Eq. 2. These points are shown in Fig. 40 and are compared with a curve plotted from the average values for K taken from Table 12. The curve divides the failed from the satisfactory fields in a reasonable manner. The fact that in one case a point indicating a failure falls within the satisfactory zone indicates that the curve is not unduly conservative. It also is clear that the shape of the curve for CBR values greater than 15 has not been fully substantiated.

Throughout this analysis no value has been assigned to the surface contact pressure; it has been designated simply as a constant. Messrs. Middlebrooks and Bertram mention a pressure of 60 lb per sq in. for their extrapolations and it is known that contact pressures at the Stockton test section No. 2 exceeded 100 lb per sq in. The contact pressures reported for the airfields and the test sections listed in Table 9 varied between approximately 50 lb per sq in. and 100 lb per sq in. In view of the fact that the curve presented in Fig. 40 is applicable to the loading conditions represented by the plotted data, it is considered that this curve is valid for contact pressures up to and including at least 100 lb per sq in.

RAUL VALLE-RODAS,[55] ASSOC. M. ASCE.—Among the various methods for the design of flexible pavements, the one developed by the California Division of Highways has proved to be very practical and reliable. It is being used successfully in Central and South America.

One of the advantages obtained is by using the California compaction method, with an applied static load of 2,000 lb per sq in. This method gives a low moisture content for the point of maximum density, which facilitates greatly the compaction work in the field. On the other hand, it was found that the "optimum moisture" obtained by using the standard method (the method of the American Association of State Highway Officials (AASHO)) is sometimes at or near the plastic limit. At this point the soil is too wet and cannot be compacted properly.

Likewise, the following results (which are in close agreement with the ones reported in the Symposium) have been obtained:

1. There is no definite relationship between the modified AASHO and the California compaction method. In most of the cases higher values (up to 6%), for maximum dry densities, were obtained from the California static method. Also, the California bearing ratio (CBR) varied appreciably in both cases.
2. In general, CBR varies noticeably with density. Therefore, it appears to be more reasonable to base the design of flexible pavements on the CBR corresponding to the density actually obtained in the field after compaction, rather than on the maximum density determined in the laboratory.

[55] Cons. Engr., Materials Testing Dept., Ministry of Public Works, Caracas, Venezuela.

3. Between the CBR values at 0.1 in., and the expansion obtained after soaking the sample for 4 days, the following relationship has been established:

(*a*) Soils that expanded 3% or more had CBR values of less than 9%.
(*b*) Soils having a CBR equal to or higher than 15% showed expansions equal to or lower than 2%.
(*c*) Soils having a CBR higher than 30% expanded less than 1%.

4. In many cases the stress-penetration curve is not a smooth one. If the test is rerun and the same nonsmooth curve is obtained again, it appears to be more logical to base the design of flexible pavements on the average CBR at 0.1 in., 0.2 in., 0.3 in., 0.4 in., and 0.5 in., rather than at 0.1 in., or 0.2 in.

In general, the selection of the CBR value for design purposes should be based on field conditions, and readjustments should be made according to the density, moisture, depth at which the water table is found, rainfall, drainage conditions, and other existing conditions.

TOM W. STALLWORTH,[56] ASSOC. M. ASCE.—Information clarifying a number of points that have not been understood by either the proponents or the opponents of the California bearing ratio (CBR) method for airfield pavement design is contained in the Symposium.

In the papers by Messrs. McFadden and Pringle, Porter, and Middlebrooks and Bertram, the point is made that a form of shear test is required in the design of flexible type pavements and that the CBR test is a measure of shearing resistance of soils. From a knowledge of the test details it would seem that the resistance of the soil sample to penetration of the piston consists of two reaction components comparable to those which resist the penetration of a friction pile. These components are (1) the resistance to punching shear around the periphery of the piston and (2) the point resistance which is distributed over the area of the contact face of the piston. Since the modulus of shearing resistance is determined by the resistance at 0.1-in. penetration, it probably can be argued that the first of the components is responsible for a predominant part of the resistance to penetration for such a low value of deformation where the second component has not yet been mobilized to a significant degree. This explanation seems logical if the soil is a cohesive material; but the value of the modulus of shearing resistance obtained in this manner, for coarse-grained base materials that are of very low compressibility, would appear to be a measure of penetration resistance, not shearing resistance. The question arises relative to the limitation as to type of soil for which the test is valid. It is understood that the California Division of Highways does not use the test to determine a measure of the shearing resistance of all types of soils, including flexible base materials, and some discussion of this fact in the closure will be appreciated.

In the description of the adaptation of the CBR method to the design of airfield pavements the extrapolation of the original curves has been made on the basis of the elastic theory. It is significant that shear stresses were used

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as a guide in making the extrapolation and, for the first time to the writer's knowledge, the pressure used in this process is stated.

Published data on airfield test projects indicate that tire inflation pressure and tire contact pressure are not exactly the same. Therefore, it would be more accurate to state that contact pressure of 60 lb per sq in. was used in the extrapolation because that was the approximate value of the tire inflation pressure.

Fig. 6 illustrates a fact which the authors recognize, but which has not received the attention it deserves in limiting the use of the design curves. This figure shows that the maximum shearing stress for the 70-kip wheel load is no greater than that produced by a 12-kip load. If the contact pressure remains constant the increased load is reflected in an increased bearing area which does not increase the magnitude of the maximum shearing stress but does increase the depth at which it occurs. The important part of this discussion is that, unless the qualifying contact pressure is given, the designer is not aware of the effect of contact pressure and considers gross wheel load alone as the significant factor in the design. The trend in inflation pressures for aircraft tires is upward to a marked degree and the maximum shearing stress of 38 lb per sq in. induced by a 12-kip load at a contact pressure of 120 lb per sq in. is double the maximum shear stress that develops when this same load is applied at a pressure of 60 lb per sq in. This increase in shear stress resulting from changes in aircraft landing gear design probably will require some modification of the empirical design curves now in use.

This deficiency of the CBR design curves has been noted independently by several writers, but it is believed that the implications of greatly increased contact pressures have not reached the engineers who are responsible for pavement design.

ROBERT HORONJEFF,[57] ASSOC. M. ASCE.—The eleventh Symposium paper by Mr. Turnbull is an excellent summary on the advantages and shortcomings of the California bearing ratio (CBR) method. The writer wishes to elaborate on some of the points mentioned by Mr. Turnbull. The comments are not to be considered as criticism of the CBR method but are merely presented to make the practicing engineer fully aware of certain phases of the method.

As indicated in the fourth Symposium paper, CBR values are extremely sensitive to the density of the soil and this fact is illustrated in Fig. 11(*c*). With a moisture content of 6% and a molded dry density of 120 lb, the CBR value is approximately 38. If the density is increased to 125 lb, the CBR of the same material is increased to 78, or more than double the original value. Extreme variations also occur in the highly plastic soils with CBR values of from 3 to 10. In one airport the writer found that the CBR value was 6 when the dry density was 106 lb per cu ft; however, when the density was increased to 115 lb, the CBR was increased to 20. Since there is no evidence that precise correlation exists between the compaction of a given soil by any laboratory procedure and the compaction of the same soil in a fill, the density of the fill could easily vary as much as 5 lb per cu ft from the laboratory compacted sample.

[57] Prin. Engr., Corps of Engrs., and Lecturer in Transportation Eng., Univ. of California, Berkeley, Calif.

Referring to the design curves in Fig. 7, paper No. 3, it is readily seen that small variations in CBR values for subgrade soils with CBR values of 30 or greater will not result in appreciable differences in the total thickness of pavement and base required to support a specified wheel load. However, the same is not true for subgrade soils whose CBR values range from 3 to 10.

To illustrate this point further assume that a flexible pavement is to be designed for a wheel load of 70,000 lb. If the CBR of the subgrade is 38, the required thickness of pavement and base is 10; however, if the CBR is increased to 78, the thickness of pavement and base is reduced to 6 in. In this instance only a 4-in. variation in thickness results. Now, assume that the CBR of the subgrade soil is 3. The total thickness of pavement and base required for a 70,000-lb wheel load is 50 in. If the CBR value is increased to 5, the total thickness of pavement and base is reduced to 36 in. In this case the CBR changed by only 2 but the thickness varied approximately 14 in. It should be remembered that 2% is often not within the accuracy of the test.

From the foregoing discussion it can be readily seen that the CBR values for the weaker subgrade soils must be established with greater care and accuracy for design purposes than for soils with a CBR of, say, 25 or greater. The design CBR for airports is usually established from explorations and laboratory tests prior to initiation of construction. It is often very costly to revise the thickness of pavement and base after construction has begun, since it would normally delay the contractor until revised grades could be established. For this reason extreme care must be taken to insure that the density and moisture content of the laboratory sample conform as closely as possible to the density and moisture content of the fill in place. Most of the time this is difficult to achieve.

Another point which the writer desires to elaborate on is the 4-day soaking period. It is gratifying to know from Mr. Turnbull's discussion (paper No. 11) that

> "* * * the Corps [of Engineers] is making a serious effort to determine in what general areas and specific localities the conservatism of design as represented by the 4-day soaking period need not be followed.".

In several instances the writer found that the moisture content of the subgrade several years after completion was less than the moisture content of the corresponding laboratory soaked samples. It is recognized that in many parts of the United States this condition would not be true. Furthermore, the moisture content can vary considerably throughout the year depending on the season. However, in the interest of economy, consideration should be given to modifying the soaking period in localities where it can be definitely established that the moisture content will not vary appreciably from conditions at the time of construction—especially for the soils where a small variation in CBR results in large differences in total thickness of pavement and base. This is well illustrated by the data contained in Table V of the paper by Norman W. McLeod on load testing applied to pavement design.[50]

[50] "A Canadian Investigation of Load Testing Applied to Pavement Design," by Norman W. McLeod in "Symposium on Load Tests of Bearing Capacity of Soils," *Special Technical Publication No. 79*, A.S.T.M., 1947.

JOHN J. O'MARA,[58] ASSOC. M. ASCE.—In the last and summary paper of the Symposium Mr. Turnbull relates that considerable criticism has been made of the procedure for the preparation of the test specimens in that the 4-day soaking period is excessive and leads to pavement thicknesses which are too conservative. One aspect of the comparison of the degree of saturation of the laboratory sample to that of the prototype has not been discussed fully. The spring breakup period in many sections of the United States is the critical point of saturation for thousands of miles of airfield and highway pavements. During this period (which occurs in all regions of moderate to deep frost penetration) the ice lenses melt and leave the subgrade in a saturated or, perhaps, a "supersaturated" (excess hydrostatic pressure) condition. The reduction in shearing resistance due to such moisture conditions is much more severe than that created by the 4-day soaking period specified for the laboratory specimens because the latter rarely results in degrees of saturation approaching 100%.

In view of the conditions prevalent under pavements as frost leaves the ground, it would be well to continue the specified soaking of laboratory specimens of subgrades in frost regions until such time as experience and the long-range studies referred to by Mr. Turnbull develop qualifying procedures. In this connection it is suggested that caution be exerted in comparing pavement performance during a few springs with the required thickness of pavements. First, critical spring breakup periods occur about only once in every 5 years, and with no regularity. Second, pavement failure during the period is caused by a combination of the moisture weakened subgrade and the application of heavy loads on the pavement. On major highways the loads coincide with the subgrade condition; on airfields, repetitive wheel loads are distributed over a wide area, and operations are limited to a few repetitions per hour or may be suspended during the critical period. Consequently, an airfield pavement may not fail during the spring breakup for several years, and judgment should be withheld until sufficient time has elapsed to evaluate the pavements properly.

The writer questions the unqualified rule of adjusting the stress-penetration curves to correct initially concave upward curves. It is true that it is practically impossible to produce a stress-deformation curve of any kind which begins at the origin and has the correct shape. For this reason most materials testing requires the application of a small load before extensometers are attached and set. In some cases this is not feasible and the stress-strain curve is adjusted to correct for uneven bearing and other faults at the beginning of the test. However, these adjustments amount to no more than one or two thousandths of an inch in ordinary materials testing. In a test conducted by the writer in May, 1949, the adjustment amounted to more than 0.1 in. and resulted in a CBR number of 50 compared to a CBR of 10 for the unadjusted curve. Some factor other than uneven seating of the piston must be involved in such an adjustment. This may be the actual shearing deformation prior to consolidation of the soil immediately below the piston. If so, it is of importance and should not be disregarded.

However, this deformation may be due to eccentric or other inconsequential movement occuring between the frame of the testing machine and the top of

[58] Asst. Prof. of Civ. Eng., State Univ. of Iowa, Iowa City, Iowa.

the mold. Some research should be devoted to arranging the strain gage to measure the deformation of the soil, not the movement between the frame and a point on the mold.

Until such a procedure is developed, better data could be secured by using two strain gages, in the same fashion as the present one, but arranged to record the movements of diametrically opposed points on the rim of the mold. The readings of the gages, then, should be averaged to secure the penetration. Still, caution should be exerted in any adjustment of the resulting stress-penetration curve, and a limit should be set for such adjustment—that is, a maximum offset from the original origin of this curve.

D. P. KRYNINE,[59] M. ASCE.—An elementary design rule requires that a structure must be safe and economical. In its turn, an elementary principle of engineering economics states that in comparing two different solutions of an engineering problem from the economical point of view, not only should the original cost of a structure be considered, but also the capitalized cost of its repairs and maintenance during its span of life.

Pavements are structures. That flexible pavements designed using the California bearing ratio (CBR) method are safe is beyond doubt. Accelerated traffic tests and heavy war service have proved it. Sometimes it is claimed, however, that flexible pavements designed using the CBR are "overdesigned," because of the excessive soaking of samples in the test. Possibly this is true in some instances; but a record of such instances has yet to be presented. The Corps of Engineers, United States Army, who introduced the CBR during the emergency period clearly understood it. In his paper Mr. Turnbull states, "Long-range studies are being conducted to determine the moisture content that occurs under pavements in arid, semiarid, and wet regions." This initiative is commendable as a step in the right direction.

In the technical press during 1948 and 1949 statements have appeared indicating that wheel loads of any given magnitude can be carried by a pavement and base course which have only a fraction of the thickness specified by the Army design.[49] Again in some cases this is true. The writer, however, does not discern any answer to two questions that make him rather uneasy: (*a*) How long will these pavements with a fraction of the thickness specified by the Army design remain adequate? and (*b*) How long will they stay in a really good condition under heavy traffic in the case of a sudden emergency? This recalls the necessity of working out the theory of the economic life span of a runway. For many years highway people have had an analogous theory for pavements. A pavement has only a predetermined life span, after which it is more economical to replace it with a new one, instead of continuing the repairs. Only when such a theory is elaborated and put to work is it possible to determine a correct comparison between two pavements on the basis of sound engineering economics, as stated at the beginning of this discussion.

[59] Cons. Engr.; Visiting Prof. of Eng., Univ. of California, Berkeley and Los Angeles, Calif.

[49] "Airport Runway Evaluation in Canada," by Norman W. McLeod, *Research Report No. 4B*, Research Correlation Service, Highway Research Board, National Research Council, 1947.

Accumulation of Moisture Under Pavements.—Apparently few engineering questions are as controversial as the question concerning the accumulation of moisture under pavements. Some engineers claim that a 100% subgrade saturation combined with high moisture content is quite possible in all cases, whereas others state that there is a tendency for decrease in subgrade moisture under pavements with traffic and time. Both schools of thought base their opinions on field observations. Unfortunately for them, and for the profession, their observations are incomplete and reveal only a part of the true picture. At the annual meeting of the Highway Research Board, National Research Council, held in Washington, D. C., in December, 1948, L. D. Hicks, chief soil engineer of the North Carolina State Highway and Public Works Commission, read a paper entitled, "Observations of Moisture Contents and Densities of Soil Type Bases and Their Subgrades." Among other things, Mr. Hicks determined the moisture content of the subgrades of flexible pavements at twenty points (stations) of the State of North Carolina each month of the year 1947, using samples of soil spaced from 5 ft to 10 ft along the length of the highway, and taken at a distance of about 30 in. from the edges of the pavement. Presumably, his work may have only a relative value for runways that are considerably wider than highways; but his results are instructive and interesting for any pavement engineer. A few of Mr. Hicks' data follow:

Station 1.—The subgrade soil at this station was designated Wadesboro B and C (approximately A-7 in the Public Roads Administration (PRA) classification). The pavement was constructed in 1942. The observed moisture content of the subgrade was 33.0% in December, 32.2% in May, and only 21.8% in January and February. The degree of saturation in May was 96.1%.

Station 2.—This was a subgrade soil designated Colfax B (approximately A-7 in PRA classification). The pavement was constructed in 1938. In April the moisture content was 32.5% and the degree of saturation, 99.7%; and in September the moisture content was only 13.8%.

Station 3.—The subgrade soil at this point was designated Tirzah B and C (approximately A-7 in PRA classification). The moisture content in June was 26.6% with 100% saturation. The lowest moisture content value at this station was 22.7% in December.

Station 8.—The subgrade soil at this point was Lloyd B and C (approximately A-5 and A-7 in PRA classification). The pavement was constructed in 1939. The 100% saturation occurred in September (moisture content 25.9%). The lowest moisture content was in February (15.1%).

Station 13.—The subgrade soil was Wadesboro B and C (approximately A-5, A-6, and A-7 in PRA classification). The pavement was constructed in 1937. The 100% saturation occurred in January (moisture content 25.0%). The lowest moisture content was in April (19.1%).

Mr. Hicks' data show how careful one should be in interpreting field observations. Consider a hypothetical case of an investigator who in January of a certain year determined the moisture content at station 13, and came back, say, six years later at a wrong month, say, April. If seasonal fluctuations of

the soil moisture are not considered, an erroneous conclusion may be drawn that the soil moisture has decreased by about 1% per yr "due to traffic and time."

It is obvious that 100% saturation under a runway pavement may be exceedingly dangerous when a heavy plane is landing, no matter what the value of the moisture content. In such a case the total load is taken up by the pore water with a possibility of a "quick" state of the subgrade.

Moisture Under Pavements Under Desert Conditions.—Besides interesting data on soil moisture under desert conditions, gathered by the California Division of Highways, there are observations and suggestions by long time residents of desert regions. Of course, these observations require criticism and thorough analysis; their value cannot be denied, however. In a letter to the writer J. M. Lackay, district engineer of the Asphalt Institute, Los Angeles, Calif., called the attention of the writer to a case when—

> "* * * several miles of pavement in the driest part of the desert about three miles west of Salton Sea (Imperial Valley, Calif.) failed so rapidly that it could not be repaired and had to be abandoned, although a test hole put down a few feet from the edge of the pavement, some 20 or 30 feet in depth, did not disclose as much as a trace of moisture in the soil while at the same time the moisture accumulated under the pavement reduced its supporting power to that of a thin liquid."

Mr. Lackay furthermore describes an opposite case of a paved street through Balboa Park at San Diego, Calif., where an asphalt pavement about 1.5 in. in thickness laid directly on a local yellowish adobe clay was in perfect condition at the age of 14 years, whereas adjoining concrete pavements on both ends had been pounded to pieces because of the accumulation of water under them.

Thermo-Osmosis.—Engineers using the new term, "thermo-osmosis," describe the phenomenon as the natural tendency of water to flow from a relatively warm to a relatively cold region within a saturated or partly saturated soil mass. According to them, black pavements absorb much heat in warm seasons, resulting in a downward movement of soil water. To clarify the situation, the writer wishes to state that what is termed by some engineers "thermo-osmosis" is not a new phenomenon but simply a particular case of a phenomenon controlled by the second law of thermodynamics established by Carnot more than a hundred years ago. That law[60] may be formulated thus: "No self-acting device unaided by an external agency can transfer heat from a body of lower temperature to one of higher temperature."

In applying this law, its limitations must be clearly understood; and, if colder water moves to warmer regions of a soil mass, as is sometimes the case in nature, an "external agency" should be sought. Thus, when colder water moves toward hot asphaltic pavements (as apparently occurs sometimes) an "external agency" should be operative. Without field and laboratory investigations along these lines the writer hesitates to define the physical nature of such an agency even approximately. However, an analogy is that of cold soil water moving to the top of a tree, sometimes to incredible heights. This movement is caused by the strong negative pressure, developed by the leaves of the tree,

[60] "Thermodynamics of Soil Moisture," by N. E. Edlefson and Alfred Anderson, *Hilgardia*, February, 1943, p. 60.

which permits the tree to maintain a diffusion pressure of water in the roots lower than in the surrounding soil.[61]

Conclusions.—(*a*) Designers and builders of runways should consider the economic span life of pavements in comparing two types of pavements or two different pavement thicknesses of the same type; and (*b*) a thorough study of the accumulation of moisture under pavements, including the factors that possibly influence such accumulation, is an engineering problem of major importance.

[61] "Fracture of Liquids: Nucleation Theory Applied to Bubble Formation," by J. C. Fisher, *Scientific Monthly*, June, 1949, p. 415.

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DISCUSSIONS

DIFFUSION OF SUBMERGED JETS

Discussion

BY W. DOUGLAS BAINES, AND ABRAHAM STREIFF

W. DOUGLAS BAINES,[23] Jun. ASCE.—The simple, easily understood analysis of submerged jets, presented in this paper, should be of considerable value to the engineering profession. The writer assisted in the evaluation of some of the mathematical expressions and subsequently extended the study of axisymmetrical jets by investigating the Reynolds effect and the turbulence. On the basis of the material gained in this study, revaluation of certain results which the authors present appears to be warranted.

The authors' assumption that the Reynolds effect is negligible is unnecessary because the basic assumptions of the analysis (constant pressure throughout, similarity of velocity profiles, and constant spread of the jet) do not specifically involve the Reynolds number. The analysis should describe the flow at any given Reynolds number where the assumptions are satisfied. Thus, it would appear that the method of attack of the general problem would be to investigate the flow pattern at several different Reynolds numbers and to note in what ways the pattern changes as the Reynolds number changes. Of the three main assumptions, the first (constant pressure) has been checked theoretically and found satisfactory by W. Tollmien,[5] and the other two have been corroborated by the writer. The similarity of velocity profiles for a constant Reynolds number is shown in Fig. 24, which gives the velocity profile for a different Reynolds number.

The corresponding velocity distributions can be expressed empirically as follows: For $\mathbf{R} = 2.1 \times 10^4$—

$$\frac{v_x}{v_{\max}} = \exp\left[-43.3\left(\frac{r}{x}\right)^{1.82}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (82a)$$

NOTE.—This paper by M. L. Albertson, Y. B. Dai, R. A. Jensen, and Hunter Rouse was published in December, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1949, by James S. Holdhusen, Duilio Citrini, and Stanley Corrsin.

[23] Research Associate, Iowa Inst. of Hydr. Research, State Univ. of Iowa, Iowa City, Iowa.

[5] "Berechnung turbulenter Ausbreitungsvorgänge," by W. Tollmien, *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 6, 1926, p. 468.

and, for $\mathbf{R} = 7 \times 10^4$—

$$\frac{v_x}{v_{max}} = \exp\left[-64.4\left(\frac{r}{x}\right)^{1.84}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (82b)$$

These curves, plotted through observed data, are shown in Figs. 24(*a*) and 24(*b*), respectively. The linear spread of the jet is substantiated in Fig. 25,

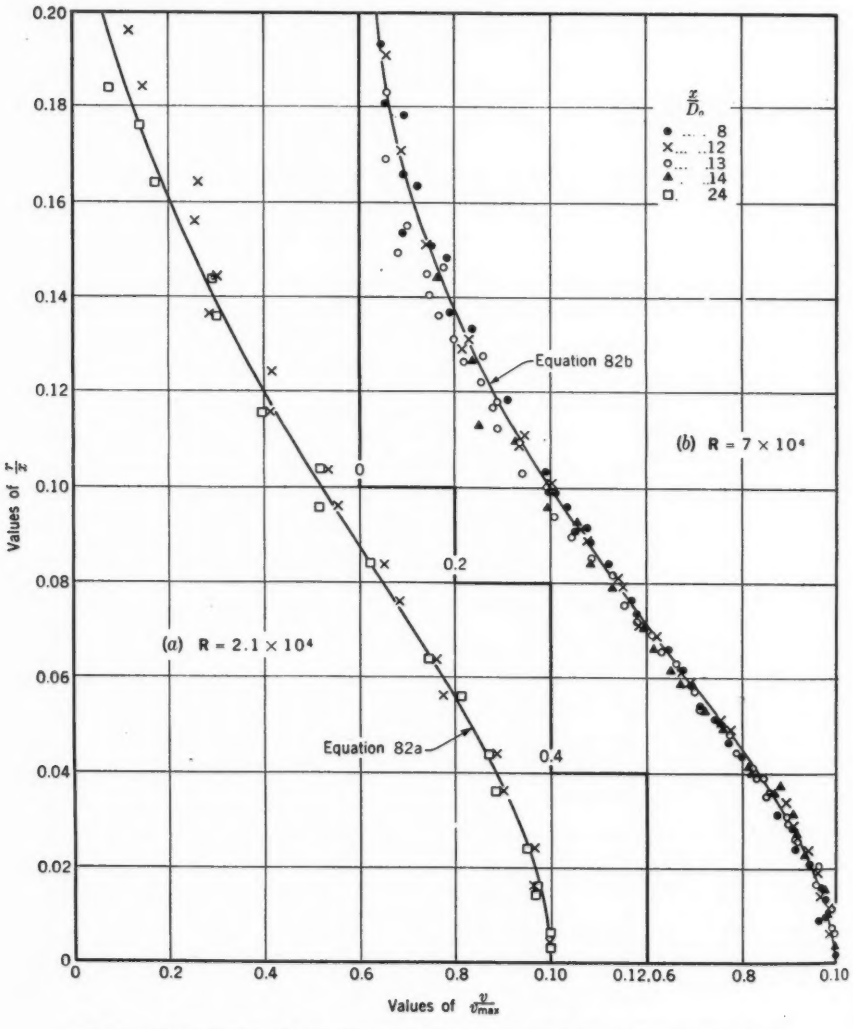

FIG. 24.—DISTRIBUTIÓN OF LONGITUDINAL VELOCITY IN THE ZONE OF ESTABLISHED FLOW

a plot similar to Fig. 15. Again, for a constant Reynolds number the assumption appears to be satisfied.

A more rigorous mathematical analysis, assuming a general velocity profile, also results in Eqs. 23, 25, 27, 28, 30, and 32, thus illustrating the fact that the

probability function is not a necessary assumption in the analysis. However, in the general case, the constants C_1 and C_2 are more complicated and thus not so convenient in practical problems as those derived with the probability distribution. The main part of such an analysis is given by H. W. Liepmann and J. Laufer[11] for the two-dimensional jet and by the writer for the three-dimensional case.[24] This analysis shows that the bulk properties of the flow can be described by the constant x_0/B_0 (or x_0/D_0) as is shown in the special case by the authors. This constant, the length of the potential zone, is a physical property of the jet that is easy to visualize and is perhaps preferable to C_1 or C_2, the constant appearing in the equations. Thus, extending the reasoning that the analysis describes the jet at any given Reynolds number, the effect of changing the Reynolds number should be described by the variation of x_0/B_0 (or x_0/D_0).

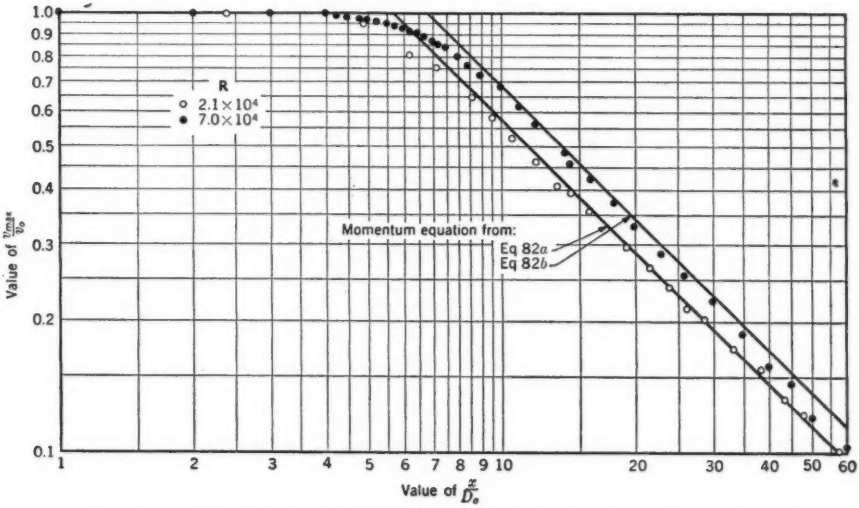

FIG. 25.—DISTRIBUTION OF CENTER-LINE VELOCITY FOR R = 2.1 × 10⁴ AND 7.0 × 10⁴

Experimental verification of the foregoing reasoning is given by Fig. 26, in which the length of the potential zone is plotted as a function of the Reynolds number. The data in Fig. 26(*a*) were obtained using a blower to furnish the air stream, for which the intensity of turbulence, $\frac{\sqrt{[(v')_x]^2}}{v_0}$ was 0.008 at a Reynolds number of 7×10^4. The fluctuations were not completely random, because the blades of the blower induced a dominant turbulent frequency of 360 cycles per sec. The regularity of these pulsations could easily be noted by placing the output of a hot-wire anemometer across an oscilloscope and varying the sweep frequency. The data in Fig. 26(*b*) were obtained with the

[11] "Investigations of Free Turbulent Mixing," by H. W. Liepmann and J. Laufer, *Technical Note No. 1257*, National Advisory Committee for Aeronautics, Washington, D. C., August, 1947.

[24] "Investigations in the Diffusion of Submerged Jets," by W. D. Baines, thesis presented to the State University of Iowa, at Iowa City, Iowa, in August, 1948, in partial fulfilment of the requirements for the degree of Master of Science.

air line used at the State University of Iowa, in Iowa City, as the source of air. The pressure in this line was maintained constant at 100 lb per sq in. by motor-driven compressors. The intensity of turbulence in the jet mouth in this case was 0.004 at a Reynolds number of 7×10^4 and appeared to be randomly distributed. The effect of neglecting viscous shear should be less as the Reynolds number increases, and, hence, the data in these two figures

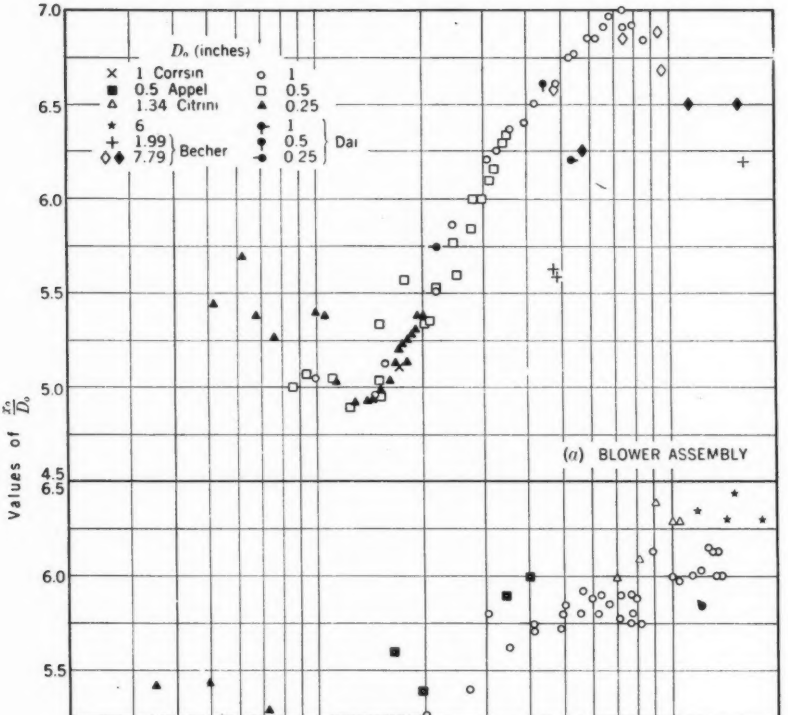

FIG. 26.—VARIATION IN THE LENGTH OF THE POTENTIAL CONE WITH THE REYNOLDS NUMBER

should approach an asymptote at large values. In Fig. 26(*b*) the data appear to be doing so, but the trend is not definite enough to be conclusive. No asymptote could be drawn in Fig. 26(*a*) as limitations of the blower prevented extension into this range. The data in Figs. 26(*a*) and 26(*b*) agree for low Reynolds numbers; but above $\mathbf{R} = 2 \times 10^4$ there is a steadily increasing deviation. A longer potential cone was obtained with a blower than with the air

line. This may be the result of the regular fluctuations forming ring vortices at the mouth of the jet. As ring vortices tend to persist, they would prevent the formation of smaller, random eddies in the zone of high shear. With the air line, the spread of the jet should be caused by eddies of random size and occurrence forming in the high shear zone.

The effects of the Reynolds number and the turbulence of the issuing stream can be noted in the authors' results and those of other investigators. The results presented by the authors, by S. Corrsin,[10] D. Citrini,[21] Poul Becher,[25] and D. W. Appel,[26] and by tests of the velocity distribution downstream from a 6-in. bellmouthed opening mounted in the small wind tunnel of the Iowa Institute of Hydraulic Research, at Iowa City, are plotted in Fig. 26 for comparison. If regular fluctuations could be expected, the results would be plotted in Fig. 26(*a*); if not, they would be plotted in Fig. 26(*b*). It may be noted that Mr. Corrsin also observed the regular fluctuations in the velocity at the jet mouth, presenting an oscillograph,[10] similar to that which the writer observed. Considering the scatter of the writer's data and the unknown turbulence conditions in the other cases, the agreement between various investigations is surprisingly good. All the investigations show the definite lengthening of the potential cone with an increasing Reynolds number.

The authors' assumption of constant momentum flux was corroborated by the writer's measurements. This is illustrated by computing the length of the potential cone from the velocity profiles, Fig. 24, using Eq. 6, and comparing it to the result obtained by direct measurement. This computation gave values of 5.76 and 6.90 from the empirical curves in Figs. 24(*a*) and 24(*b*), respectively. These values are close to the values obtained from Fig. 25, which are 5.7 and 7.0 for corresponding Reynolds numbers. It should be noted that the scatter of the experimental points in Fig. 17 is a result of the Reynolds effect. The lack of this scatter in the writer's data thus makes the corroboration of the momentum assumption more definite.

The turbulent properties of the flow are of great theoretical importance because they give an insight into the general turbulent flow of fluids. In the authors' simple analysis of the turbulence, the equations relating the shear to the mean flow pattern—Eq. 35 and the corresponding one for axisymmetric flow—are fundamental. It is anticipated, therefore, that good agreement between the properties derived by these equations and the experimental values should exist. In Fig. 27 the measurements of turbulent shear which the writer made with a hot-wire anemometer are presented. These consist of traverses at $x/D_0 = 12$ and $x/D_0 = 20$ and indicate that there is similarity of shear profiles in this zone. The experimental points, however, do not follow the shear profile computed by the equation corresponding to Eq. 35. This large

[10] "Investigation of Flow in an Axially Symmetrical Heated Jet of Air," by S. Corrsin, *Advance Confidential Report No. 3L23*, National Advisory Committee for Aeronautics, Washington, D. C., 1943.

[21] "Nuove richerche sulla diffusione di una vena liquida in un campo di liquido in quiete," by Duilio Citrini, *L'Energia Elettrica*, June, 1947.

[25] "Om beregning af Indblaesningsåbiniger," by Poul Becher, I Kommission Hos Jul. Gjellerups Forlag, Copenhagen, 1949.

[26] "The Development of a Turbulence Pitot for Use in Water," by D. W. Appel, thesis presented to the State University of Iowa, at Iowa City, Iowa, in June, 1949, in partial fulfilment of the requirements for the degree of Master of Science.

discrepancy which also has been noted by Mr. Corrsin,[10] was probably the result of a combination of inaccuracies in the mechanical setup and in the evaluation of calibration constants in the writer's experiments.

A clearer understanding of the turbulence may be gained by considering it from the viewpoint of two of the turbulence theories. The authors' use of the

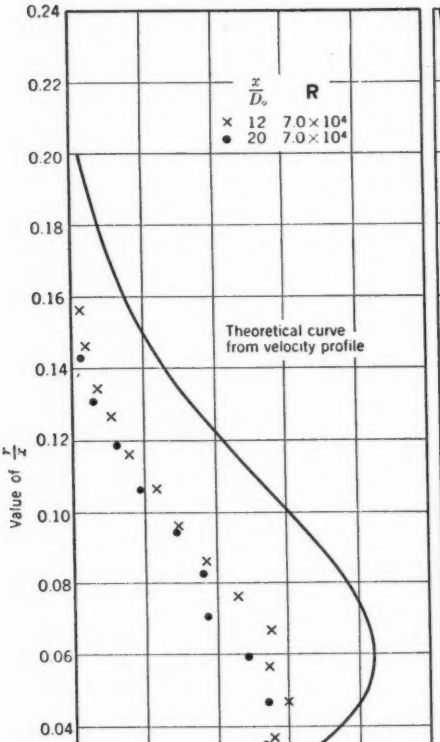

FIG. 27.—DISTRIBUTION OF TURBULENT SHEAR IN THE ZONE OF ESTABLISHED FLOW OF AN AXISYMMETRICAL JET

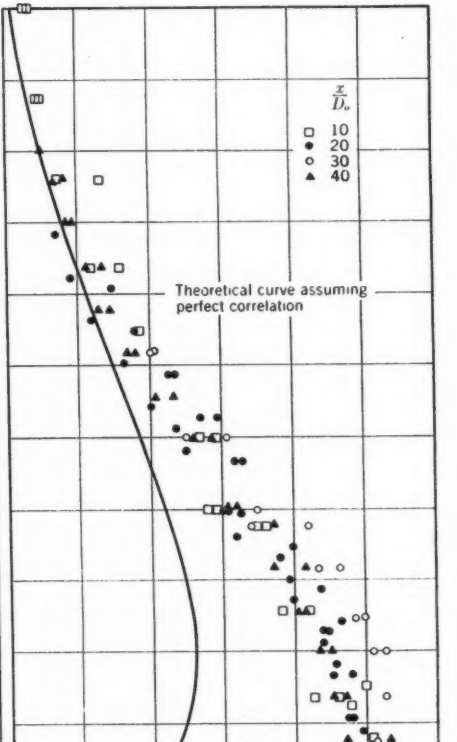

FIG. 28.—DISTRIBUTION OF TURBULENCE REFERRED TO THE CENTER-LINE VELOCITY IN THE ZONE OF ESTABLISHED FLOW FOR R = 7.0 × 10⁴

Prandtl mixing-length concept is not of practical significance since the quantities derived cannot be measured directly. Consideration of the problem in the light of the statistical theory of turbulence is not too satisfactory since the properties of the turbulence in cases of nonisotropic flow do not naturally follow the same definitions as are being used in isotropic turbulence. The intensity

and shear have the same definition in each case but have three nonequal components in nonisotropic flow. A mixing length defined as the area under the correlation curve is difficult to use for two reasons: (1) At present, correlation coefficients cannot be measured by the double-wire, hot-wire technique unless both wires are in the same mean velocity zone; and (2) the abscissa of the correlation curve should follow either the fluid particle in its motion or a path perpendicular to it. In the first case, this would mean following a streamline, which is usually a difficult procedure. Evaluation of the eddy viscosity, defined as the coefficient of diffusion at a point, suffers from the same lack of natural coordinates. A quantity that would be of great importance, and would retain the same definition, is λ, the size of the smallest eddies.[27] The size and distribution of λ would indicate the manner and rate of dissipation of turbulent energy. Unfortunately, a satisfactory method of measuring this quantity was not available to the writer.

The qualitative pattern of the turbulence can be deduced from oscillographs, however. Upon moving through the shear zone lengthwise down the jet, it was noted that the frequency of the turbulence was very high near the mouth, and that it decreased as the distance increased. A traverse taken radially outward from any point in the zone of established flow illustrated the transition of the flow from turbulent to laminar. This transition[10] progresses from pure turbulent flow in the central core of the jet, through an annular transition zone in which the flow is neither laminar nor turbulent, to a region of laminar flow that extends to infinity. The oscillograms in these zones are sharp-peaked waves of high frequency passing to more rounded wave forms of longer periods, finally becoming slow, easy undulation in the laminar regions.

The authors' assumption that v'_x and v'_r are proportional in deriving the curve of $\sqrt{(v')^2}$ in Fig. 20 from the velocity profile does not seem valid for the following reasons: A direct result of this assumption is that there is similarity in the radial distribution of the intensity of turbulence. Inspection of Fig. 28 shows that similarity of intensity profiles is a fact only when $x/D_0 > 20$, whereas the velocity profiles are similar for $x/D_0 > 8$. Making use of the foregoing assumption and the shear profile of Fig. 27, the theoretical curve (similar to the curve in Fig. 20) plotted in Fig. 28 was obtained. In order to agree with the experimental points, v'_r would have to be about four times v'_x. Thus the assumption of perfect correlation does not seem valid since measurements[10] of these quantities have not borne out this proportionality.

Problems requiring a detailed examination of submerged jets have confronted the civil engineer only recently. For many years a rough approximation of the characteristics of jets was sufficient for design purposes; but in recent times the design of stilling basins for large dams has necessitated a detailed study of the properties of jets. This has been accomplished mainly by hydraulic models, as has the companion problem of the scour caused by high-velocity flows. It should be noted that the latter is simply a study of the effects of submerged jets on sediment. Since a reasonable and workable design of a structure is possible only when the engineer is intimately acquainted with the

[27] "Statistical Theory of Turbulence," by G. I. Taylor, *Proceedings*, Royal Soc. of London, Series A, Vol. 151, 1935, pp. 421–478.

phenomena involved, this paper should greatly assist the designer of the aforementioned types of structure. In the usual case, the Reynolds number is of the same order as those in the experimental investigations reported by the authors, so that the designer is free of a further assumption. Thus, it is the writer's opinion that this simple, direct analysis of submerged jets should be of great assistance to engineers in the future.

Acknowledgment.—The investigations reported herein were conducted at the Iowa Institute of Hydraulic Research under the auspices of the Bureau of Ships and the Office of Naval Research of the United States Navy, and the Department of Mechanics and Hydraulics of the State University of Iowa.

ABRAHAM STREIFF,[28] M. ASCE.—Submerged jets of water are of practical importance because of their use in underwater spilling through dams and powerhouses. For this reason large-scale tests were made at the Croton Dam on the Muskegon River in Michigan in 1915. Based on the results of these tests, the Mio, Alcona, Hodenpyl, and Hardy dams on the Ausable and Muskegon rivers in Michigan have been provided with underwater spilling tubes. The northern part of the southern peninsula of Michigan is covered with glacial drift, which causes fairly uniform runoff conditions. The application of spillway tubes through the powerhouse therefore enabled a reduction in cost by entirely or partly eliminating an overflow spillway.

Considering that the tests reported in this paper were made on a very small scale and with air, it is not surprising to find that tests with water jets, having a velocity of 45 ft per sec issuing from nozzles from 12 in. to $20\frac{1}{2}$ in. in diameter and discharging with the top 17 in. under the water surface, would give results wholly different from the authors' findings. Instead of the Reynolds number $\frac{V\,a}{\nu}$ the Froude number $\frac{V^2}{a\,g}$ is used, because of the close proximity of a free surface, thus calling in the influence of gravity.[29,30]

The pattern of flow resulting was wholly different from that reported in the paper. The jet diffusion shown in Fig. 1 (rectilinear diffusion under a constant angle) was completely absent. Instead, a flow pattern appeared which was quite similar to the well-known streamline pattern into a Borda mouthpiece:

$$u = x + e^x \cos y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (83a)$$

and

$$v = y + e^x \sin y \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (83b)$$

This is shown plainly in Fig. 29(*a*), which shows discharge (no baffle) from a 12-in. nozzle, with the gate fully opened. The water surface elevation was 93.2; the elevation of the top of the nozzle, 91.8. The effect of the wind in producing ripples away from the nozzle is apparent.

The conclusion that "All results are considered applicable for design purposes to the flow of any liquid * * *" (under the heading, "Conclusions")

[28] Cons. Eng., Austin, Tex.

[29] "Applied Hydro and Aeromechanics," based on lectures of L. Prandtl, by O. G. Tietjens, translated by J. P. Den Hartog, McGraw-Hill Book Co., Inc., New York, N. Y., 1st Ed., 1934, p. 10.

[30] "Hydrodynamics," by Horace Lamb, Univ. Press, Cambridge, England, 6th Ed., 1932.

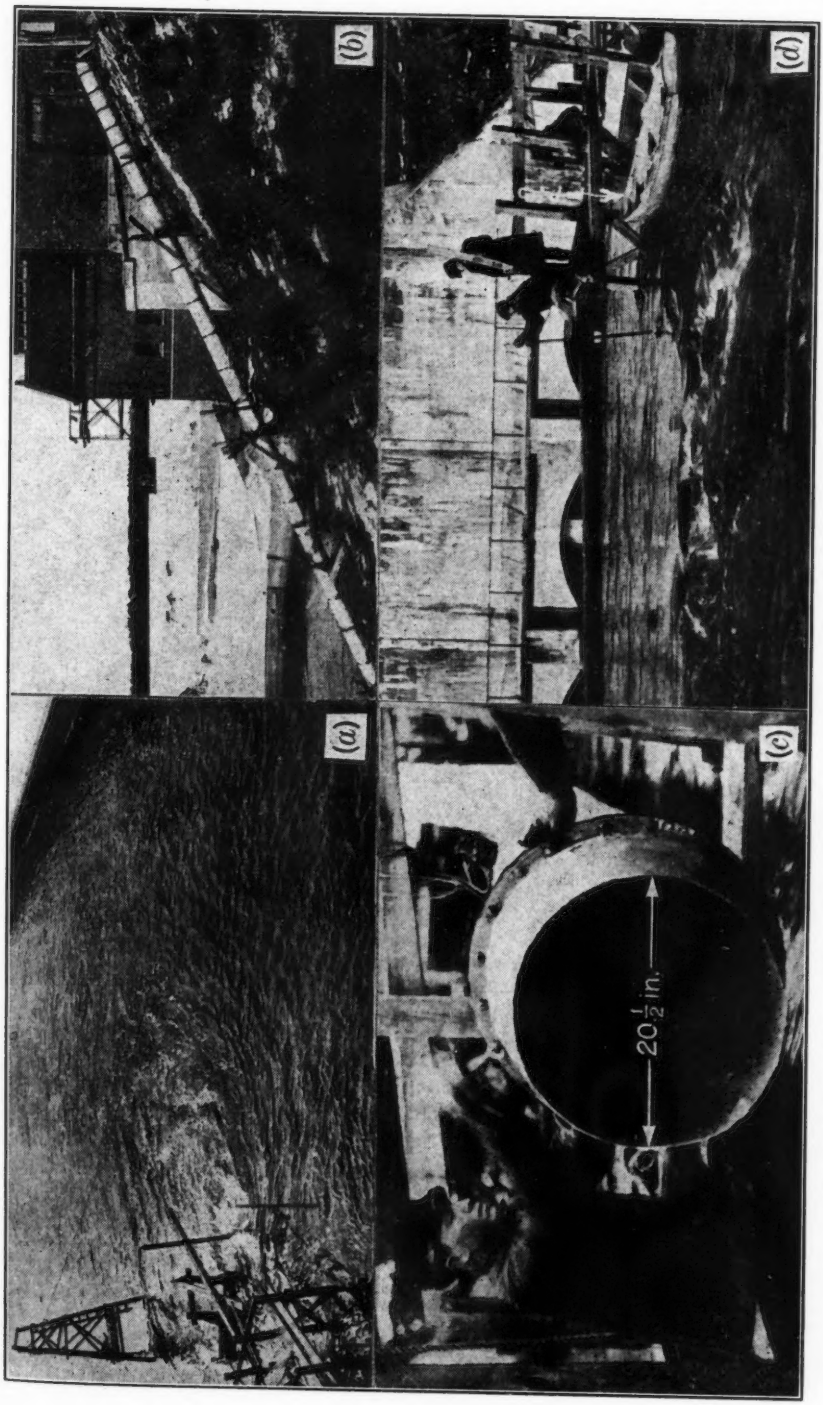

FIG. 29.—LARGE-SCALE TESTS AT CROTON DAM, ON THE MUSKEGON RIVER, IN MICHIGAN, JANUARY 29–30, 1915

is not confirmed. The small scale, the use of air instead of water with the resulting scale effect, and the absence of a free surface contribute to the complete dissimilarity of the results.

The large-scale tests were made by siphoning a 30-in. steel penstock over an earth dam adjoining the powerhouse of the Croton Dam. The nozzles varied in diameter from 12 in. to 20½ in. and operated under a gross head of 40 ft. This is shown in Figs 29(*b*) and 29(*c*). The elevation of the nozzle was such that the jet became submerged about 1 ft because of the rise in tailwater after the units in the powerhouse were opened wide.

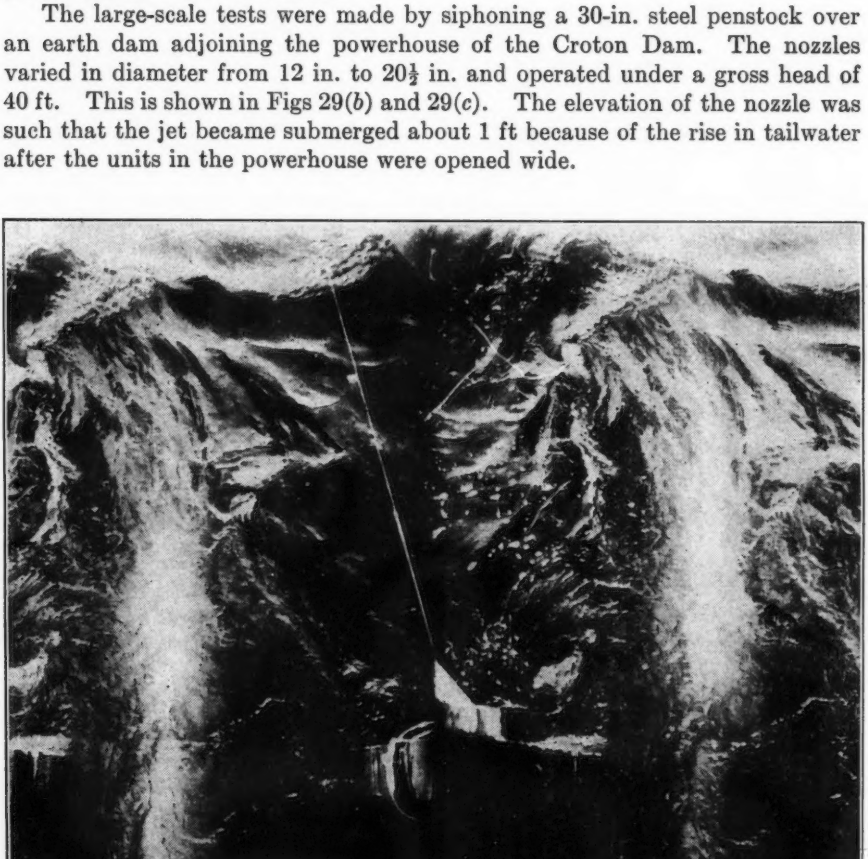

FIG. 30

The theory proposed by the authors, following previous investigators such as W. Tollmien,[5] H. Reichardt,[12] P. Ruden,[6] assumes (under the heading, "Statement of the Problem") that "* * * the sole effective force [of the jet] was the tangential shear." The large-scale tests with water on Croton Dam led to opposite conclusions. There is so much kinetic energy in the jet that the tangential shear around its circumference is minimal in comparison. The

[5] "Berechnung turbulenter Ausbreitungsvorgänge," by W. Tollmien, *Zeitschrift für angewandte Mathematik und Mechanik*, Vol. 6, 1926, p. 468.

[12] "Gesetzmässigkeiten der freien Turbulenz," by H. Reichardt, *Forschungsheft 414*, Verein Deutscher Ingenieure, 1942.

[6] "Turbulente Ausbreitungsvorgänge im Freistrahl," by P. Ruden, *Die Naturwissenschaften*, Vol. 21, 1933, p. 375.

writer concluded that the decelerating force is the reaction and equal to twice the head. This assumption was in satisfactory agreement with the test results. The kinetic energy is finally consumed by the tangential shear in the resulting turbulence.

The submerged jet and the hydraulic jump are the same; with rising tailwater the jump is finally converted into the submerged jet. The theory of the jump does not consider at all the sole decelerating force of the tangential shear (under the heading, "Statement of the Problem"), whereas in appearance the turbulence of the jump is greater than that of the submerged jet. Test results show that in both cases the deceleration occurs at about the same distance. It will be necessary to bring both theories into satisfactory agreement with each other, inasmuch as the only difference between the two phenomena is the elevation of the tailwater.

Fig. 30 shows a 12-in. jet issuing at a velocity of 45 ft per sec at a depth of 12 in. under the water surface, looking down on the surface. This surface was entirely unperturbed, the jet "drilling" a hole through the surrounding water. Sitting in a rowboat, the bottom of which was only a few inches over the jet (Fig. 29(*d*)), the writer could not notice any evidence betraying the close proximity of a submerged jet issuing at 45 ft per sec.

The turbine discharge, shown in Fig. 29(*d*), as smooth water upstream from the jet, flows at right angles over the jet as if it were a solid log lying in the water. The jet acts like a weir, under and over which the turbine discharge flows at right angles. There was not the slightest indication of a straight-line diffusion cone in accordance with the theory propounded in the paper. On the contrary, the spreading is curvilinear and quite rapid, in accordance with deceleration by the reaction force; the jet suddenly breaks through and spreads in all directions following approximately the pattern of the Borda mouthpiece. The authors' dimensionless graphs, therefore, are not in agreement with these tests.

These large-scale experiments were used in the design of apron protection for four hydroelectric plants which were placed on soil foundations. The largest one, the Hardy Dam, operates under a head of 100 ft and has been in operation since 1938. The submerged jets discharge under the hydraucone plate of the turbine units.

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DISCUSSIONS

INTEGRATING THE EQUATION OF NONUNIFORM FLOW

Discussion

BY WILLIAM D. MITCHELL, FRANCIS F. ESCOFFIER, GUY C. THATCHER, CHESLEY J. POSEY, AND M. E. VON SEGGERN

WILLIAM D. MITCHELL,[20] M. ASCE.—An ingenious and worthwhile addition to the procedures for integrating the equation for gradually-varied flow is presented in this paper. In many problems, of which those cited by the author are typical, the procedure is rapid, comparatively simple, and leads to results which are entirely reliable. It also is gratifying to note that, to a considerable extent, the author has conformed to the concepts and notations of Boris A. Bakhmeteff,[2] Hon. M. ASCE. This conformity seems extremely desirable, since Mr. Bakhmeteff's work is generally conceded to be both comprehensive and sound, and may well be accepted both as a point of departure for the discussion of other procedures and as a basis for the correlation of new material. In fact, the principal objective of this discussion will be to point out the basic similarity, and the specific differences, between the two methods.

Before embarking on this principal theme, a thought will be offered concerning the computation of the hydraulic exponent n. At the start of an investigation, there may be available a normal depth-discharge rating for the channel, developed from either computed or observed data. Such a rating, plotted on logarithmic coordinates, is all that is required for a graphical determination of n. Since S_0 is a constant, on logarithmic coordinates the curve for K is parallel to the curve for Q, so that the slopes are identical, and n may be determined from either, using the technique described by Mr. Bakhmeteff.[7] Similarly, $\log Q_1/Q_2$ may be used in Eq. 14*a* in lieu of $\log K_1/K_2$.

The applicability of the author's method to a rectangular channel has been verified by the writer, using as a criterion of accuracy a series of profiles observed in a 5-ft rectangular channel under carefully controlled laboratory con-

NOTE.—This paper by M. E. Von Seggern was published in January, 1949, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1949, by Philip Z. Kirpich and Paul V. Hodges; and June, 1949, by A. H. Davison, and Steponas Kolupaila.

[20] Hydr. Engr., U. S. Geological Survey, Champaign, Ill.

[2] "Hydraulics of Open Channels," by Boris A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932.

[7] *Ibid.*, p. 84.

ditions.[21] The proposed method yields results that are in very close agreement, not only with observed data, but also with computations by other generally accepted methods. This agreement is to be expected in view of the fact that all reliable methods have much in common. Eq. 1*c*, which may be considered the parent equation from which all the methods derive, might be written

$$dx = \frac{dy}{S_0 - S_f} - \frac{\frac{Q^2 b_w}{g A^3}}{S_0 - S_f} dy \quad \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

in which the first term on the right represents the effect of the change in depth, whereas the second term on the right represents the effect of the change in kinetic energy. In many cases, this second term has little effect on the final results; in fact, the term may often be entirely neglected in practical engineering problems. However, the entire difference between most of the reliable methods of computing profiles lies in the manner of evaluating this second term. From Eqs. 3*a*, 3*b*, and 5*b*,

$$\frac{Q^2}{g}\frac{b_w}{A^3} = \frac{M^2_c}{M^2} = \left(\frac{y_c}{y}\right)^m \quad \ldots\ldots\ldots\ldots\ldots\ldots (42a)$$

whereas, for example, by Mr. Bakhmeteff's Eqs. (45) and (47),

$$\frac{Q^2}{g}\frac{b_w}{A^3} = \frac{S_0 K^2_0}{g}\frac{g}{\sigma K^2} = \beta \frac{K^2_0}{K^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (42b)$$

Thus, to the extent to which each assumption is applicable to a given channel, there should be consistency between the results.

By a few simple substitutions, it will become evident that the methods of Messrs. Von Seggern and Bakhmeteff are identical with respect to the evaluation of the first term of Eq. 41. A careful analysis of the Von Seggern development will disclose that

$$Z = \int \frac{dy}{S_0 - S_f} = -\int \frac{du}{u^2 (1 - u^n)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (43)$$

Bearing in mind that the Von Seggern u is the reciprocal of the Bakhmeteff η, or

$$u = y_0/y = \eta^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (44a)$$

and

$$du = -\eta^{-2}\, d\eta \quad \ldots\ldots\ldots\ldots\ldots\ldots (44b)$$

—so that

$$-\frac{du}{u^2 (1 - u^n)} = \frac{d\eta}{\frac{\eta^2}{\eta^2}\left(1 - \frac{1}{\eta^n}\right)} = d\eta \frac{1}{1 - \left(\frac{1}{\eta}\right)^n} = d\eta + \frac{d\eta}{\eta^n - 1} \quad \ldots (45)$$

Using the Bakhmeteff notation (Eq. 83),

$$B(\eta) = -\int_0^\eta \frac{d\eta}{\eta^n - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (46a)$$

[21] "An Investigation of the Backwater Profile for Steady Flow in Prismatic Channels," by Wallace M. Lansford and William D. Mitchell, *Bulletin No. 381*, Univ. of Illinois Eng. Experiment Station, Urbana, Ill., March, 1949.

Eq. 43 becomes

$$Z = \int \frac{dy}{S_0 - S_f} = \eta - B(\eta) \ldots\ldots\ldots\ldots\ldots\ldots (46b)$$

which is identical to the form developed by Mr. Bakhmeteff, Eq. (90), for the case in which restoration of kinetic energy is neglected.

Now it will be obvious that Table 2, as well as Mr. Bakhmeteff's Table II, may be considered unnecessary, since the required information may be obtained from Mr. Bakhmeteff's Table IA. As an example, let $n = 3.6$, $y = 7.50$, and $y_0 = 3.00$. Then

$$u = y_0/y = 3.00/7.50 = 0.40 \ldots\ldots\ldots\ldots\ldots\ldots (47a)$$

and

$$\eta = y/y_0 = 7.50/3.00 = 2.50 \ldots\ldots\ldots\ldots\ldots\ldots (47b)$$

From Table 2, $Z = 2.464$; and, from Mr. Bakhmeteff's Table II, $Z = \eta - B(\eta) = \phi(\eta) = 2.464$. However, this result may be obtained from Mr. Bakhmeteff's Table IA, which yields $B(\eta) = 0.036$ so that $Z = \eta - B(\eta) = 2.500 - 0.036 = 2.464$ as before. Thus, Mr. Bakhmeteff's Table IA is sufficient to obtain the needed values.

The preceding statement is true, of course, only for the part of the computation that does not involve change in kinetic energy. This additional refinement, involving the evaluation of the second term of Eq. 41, is obtained differently by various investigators, and represents the differences referred to in the third paragraph of this discussion. The concept has been expressed as Eq. 12, which might also have been written as

$$\Pi = Z + \left(\frac{y_c}{y_0}\right)^m Y = \eta - B(\eta) + \left(\frac{y_c}{y_0}\right)^m Y \ldots\ldots\ldots (48a)$$

whereas Mr. Bakhmeteff's Eq. (85) might have been written in the corresponding form:

$$\Pi = \eta - (1 - \beta)\, B(\eta) = \eta - B(\eta) + \beta\, B(\eta) \ldots\ldots\ldots (48b)$$

As mentioned previously, the author's method has been found both useful and reliable for a rectangular channel. Doubtless the application may be extended to many other types of cross sections. However, the writer has been concerned with a flood plain type of cross section—that is, a cross section which, for small values of y, is comparatively narrow but which, as y increases, suddenly expands to a very great width. Several difficulties arise. Among the more serious is the fact that the rapid increase in width results in a decrease in the value of M, so that the values of m range far outside the values shown in Table 1. In fact, in extreme cases, these values of m approach negative infinity. In such cases, it appears that the usefulness of the method is confined to computing those profiles for which the normal depth is substantially greater than that required to submerge the flood plain.

Of two methods which produce equally satisfactory results, one or the other may be the more desirable from the practical viewpoint. If only the total length of reach between two widely different values of y is desired, a method

that completely integrates the varied flow equation may result in a great saving of labor. On the other hand, the length of reach between two widely different values of y cannot be taken as an absolute criterion for the comparison of two methods of computing the entire profile. Two profiles which are quite dissimilar may, by chance, intersect at two or more points. For this reason, as well as others, it may be desired accurately to define the shape of the profile for the entire reach, and the reach may need to be subdivided. Under these circumstances computations by the Bakhmeteff method may be made highly satisfactory by varying β from one to another of the subdivisions. As the desired number of points is increased, the step method, with its simplicity, flexibility, and widespread acceptance, becomes more and more attractive; but, for profiles in a channel of simple geometric cross section, for which only a few points are required, the author's method should be welcomed by all hydraulicians.

FRANCIS F. ESCOFFIER.[22]—A method for calculating backwater, which is at once precise and simple to apply, is presented in this paper. The basic assumption of the Bakhmeteff solution, that the quantity $\beta = S_0/\sigma$ (in which S_0 is the normal slope and σ is the critical slope) remains constant, is satisfactory for channels with moderate slopes. However, in steep channels where the value of β is large, the variations of that quantity can be a significant factor in determining the form of the backwater profile. The author's method permits the evaluation of that factor.

One inference that can be drawn from the use of the method is that the conventional backwater profiles shown in most textbooks are in need of modification (except for the special case $n = m$) in that they fail to show the change in character of the profiles that occur at the depth y_r for which the normal discharge is equal to the critical discharge. This depth is given by the formula:

$$y^{n-m}_r = y^n_0 \, y^{-m}_c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (49)$$

The profile shown in Fig. 12 was plotted with the help of Tables 1 and 2 for $n = 3.6$ and $m = 3.4$. The value of $(y_c/y_0)^m$ is 0.9364. It should be noted that the profile dips below the level of the pool into which the water is entering. The point of reversal in slope is in reality a point of transition from an M_1-like profile to a S_1-like profile.

In the following tabulation six such possible transition points are listed (the foregoing illustration falling in case 1):

Case 1.	$n > m \; y_r > y_0 > y_c$	M_1-like to S_1-like
Case 2.	$n > m \; y_r = y_0 = y_c$	M_3-like to S_1-like
Case 3.	$n > m \; y_r < y_0 < y_c$	M_3-like to S_3-like
Case 4.	$n < m \; y_r > y_c > y_0$	S_1-like to M_1-like
Case 5.	$n < m \; y_r = y_c = y_0$	S_3-like to M_1-like
Case 6.	$n < m \; y_r < y_c < y_0$	S_3-like to M_1-like

[22] Hydr. Engr., Corps of Engrs., Mobile, Ala.

By substituting y_r into Eq. 6,

$$\frac{dy}{dx} = S_0 \frac{1 - \left(\frac{y_c}{y_0}\right)^{nm/(n-m)}}{1 - \left(\frac{y_c}{y_0}\right)^{nm/(n-m)}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (50)$$

from which it can be inferred that $dy/dx = S_0$, provided $y_0 \neq y_c$. Thus, in cases 1, 3, 4, and 6, the slope of the profiles at depth y_r is zero. However, in cases 2 and 5, $y_0 = y_c$, and Eq. 50 assumes the indeterminate form 0/0. To

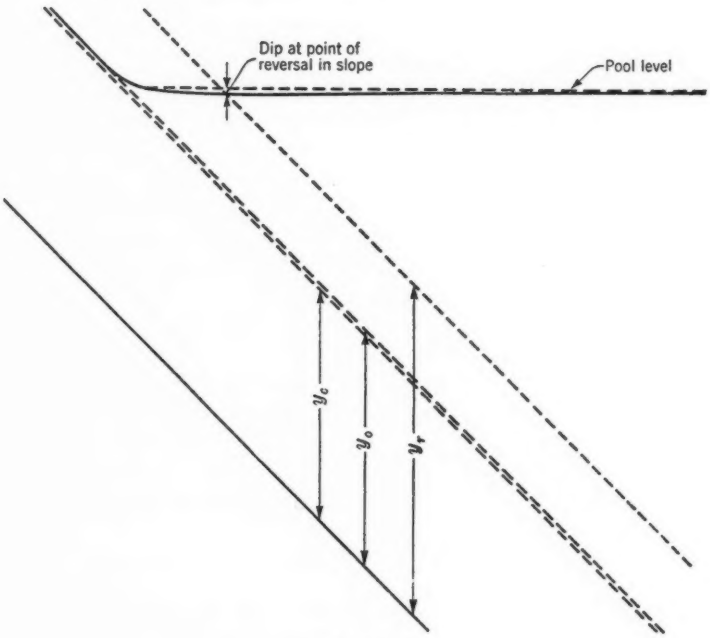

FIG. 12

determine the slope of the profile, the numerator and the denominator are replaced on the right-hand side of Eq. 6 by their derivatives. The result is

$$\frac{dy}{dx} = S_0 \frac{n}{m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (51a)$$

and the slope of the profile is therefore

$$S = S_0 \frac{m - n}{m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (51b)$$

In cases 2 and 5 the slope of the profile is not zero as is usually shown on backwater profiles but has a finite value which is minus in case 2 and plus in case 5.

GUY C. THATCHER.[23]—The integration method discussed in this paper provides what appears to be an adequate method of analysis for use with uniform channels. However, it is the opinion of the writer that the application of the method must be limited to uniform channels, and that natural channels cannot be classed as uniform in all cases. Standard methods of analysis assume that channel characteristics change uniformly from one section to the next. Thus, Mr. Von Seggern's hydraulic exponents are themselves functions of X, which means that his differential equations do not have the variables separated, and are therefore not proper for use with nonuniform channels.

Time has not permitted the writer to investigate the possibility of completing the separation of the variables in such a case, but the fact that the differential equations contain exponential functions of the left-hand variable on the right-hand side indicates that the mathematics may be extremely involved.

CHESLEY J. POSEY,[24] M. ASCE.—Together with the complete tables,[8a] this paper provides a convenient method for solving problems of nonuniform flow in channels of such cross-sectional shape that the actual variation of area and hydraulic radius with depth must be taken into account. The usual assumption is made that the friction loss varies as indicated by the Manning formula, with the rate of loss assumed equal to that in uniform flow at the same depth. Variations of friction loss and of velocity head with changing depth are quite accurately approximated in the integration, and laborious "step" computations are avoided entirely.

One stumbling block that has caused many engineers to loose interest in complete integration methods has been the necessity of taking "time out" to determine the "hydraulic exponents"—the exponents in the relationships used to approximate the variation of K^2 and M^2. Although these exponents are easily defined, their computation is often time consuming. Nagaho Mononobe's attempt at simplification[4] was unsuccessful, as it was based on the unreliable approximation of the slope of the straight line best representing a section of a curve by the mean of the slopes of the curve at each end of the section. The author's method of simplifying the determination of m and n, as shown in Figs. 1 and 2, is at once easy and theoretically sound, and has the advantage of immediately revealing how large the maximum error of the approximation may be.

The exponent m is useful not only in setting up the equation of nonuniform flow, but also in determining the critical depth. Assume, for example, that one is solving a problem of nonuniform flow in a trapezoidal channel with a bottom width of 5 ft and side slopes of 1 vertical to $1\frac{1}{2}$ horizontal. The exponent m has been determined to be 3.65 for the range of depths to be expected.

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[23] Engr., U. S. Bureau of Reclamation, Billings, Mont.

[24] Prof. of Hydr. and Structural Eng. and Acting Head, Dept. of Civ. Eng., State Univ. of Iowa, Iowa City, Iowa.

[8a] Tables 1 and 2 are greatly condensed forms of the complete tables, confined to the values of m, n, and u needed for solving the illustrative examples in the paper. The complete tables contain values of Y and Z for the ordinarily usable values of m from 3.0 to 5.0, values of n from 2.2 to 5.4, and values of u from 0 to 30. Copies of these tables are available on request at a price of $1.00.

[4] "Back-Water and Drop-Down Curves for Uniform Channels," by Nagaho Mononobe, *Transactions*, ASCE, Vol. 103, 1938, p. 950.

It is required to find the critical depth for a discharge of 200 cu ft per sec. By making use of the values of area and water surface width for a convenient value of the depth within the range, the value of the constant C_1 in the following equation is found to be 30.

$$\frac{A^3}{b_w} = C_1 y^{3.65} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (52)$$

For flow at the critical depth,

$$\frac{Q^2}{g} = \frac{A^3}{b_w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

Therefore, $\frac{\overline{200}^2}{32.2} = 30\ (y_c)^{3.65}$, from which the value of the critical depth is is 2.77 ft.

Similarly, for the channel in Example 1, the constant may be evaluated by using the value $m = 3.4$ obtained by the author and values of the area and water surface for an intermediate depth of 4 ft:

$$\frac{A^3}{b_w} = \frac{\overline{124}^3}{37} = C\ y^{3.4} = 465 \times 4^{3.4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (54)$$

Hence, the critical depth when $Q = 800$ may be found by solving for y_c in the equation:

$$\frac{Q^2}{g} = \frac{\overline{800}^2}{32.2} = \frac{A^3}{b_w} = 465\ (y_c)^{3.4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (55)$$

This yields $y_c = 3.0$, which agrees closely with the author's value of 2.98.

In obtaining the values of the exponents m and n for Example 1, the author uses a range of depths from 2.98 to 8.00. It would seem that the actual range of depths of the backwater curve, from 6.73 to 8.00, would be more appropriate, although, of course, the difference in results would be inappreciable in this instance. The actual range of depths is apparently used in Example 3.

In evaluating the over-all merit of the author's method, a comparison with the step methods in which the significant variables are rationally accounted for is inevitable.[25] In a channel of uniform cross section the step method does not involve trial-and-error solutions. However, if accuracy is to be preserved, the steps must be short, so that the labor of computation in many problems is still greatly in excess of that required by the author's method. The step method, on the other hand, has the advantage that no tables are required. Thus, it, would seem that the step method will continue to be the favorite of the occasional user but that the author's method should prove valuable to those who have many backwater curves to compute.

A question of greater importance than the comparative convenience of various methods of computing backwater curves, all based on practically the same assumptions, is that of their accuracy in representing the physical phenomena. Following the example set by Mr. Mononobe (who made tests of most of the backwater curves in flumes of different cross sections) Hsu-Hua

[25] Discussion by J. C. Stevens of "Back-Water and Drop-Down Curves," by Nagaho Mononobe, *Transactions*, ASCE, Vol. 103, 1938, p. 990.

Hu made tests of M_1-curves, M_2-curves, H_2-curves, and S_1-curves in a rectangular flume,[26] and more recently Wallace M. Lansford and William D. Mitchell, Members, ASCE, have made extensive tests of M_1-curves in flumes of rectangular and "overbank flow" type cross sections.[21] Although the sum total of experimental evidence is inconclusive, it does seem evident that the "precise" methods of computing backwater curves do not always yield results that agree well with experiment. Messrs. Lansford and Mitchell apparently found it necessary to determine the actual rating curve for the cross section and to use this information in computing their theoretical curves. This was equivalent to using a variable value of n, experimentally determined over the range of depths, a procedure not ordinarily available to the backwater curve computer. Until more systematic experimental evidence is available, it would seem that a principal use of the "precise" backwater curve methods— the author's and the step method—would be in facilitating the study of experimental evidence.

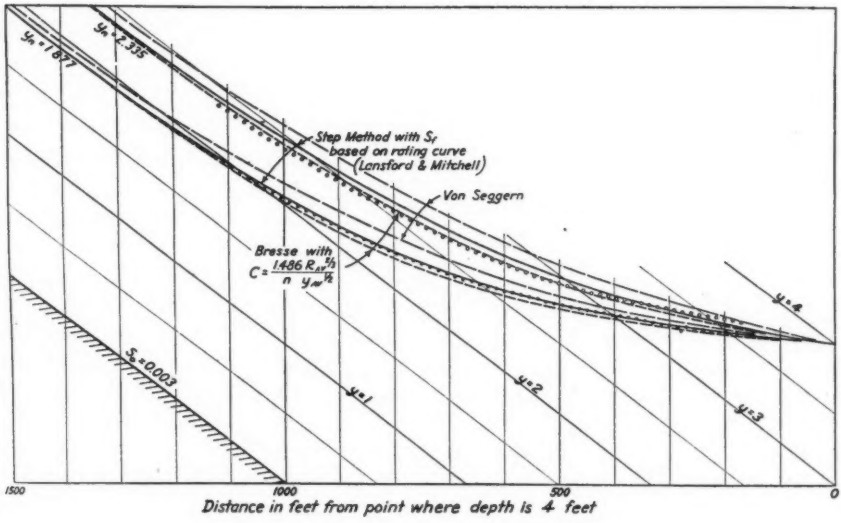

FIG. 13.—COMPARISON OF BACKWATER CURVES

For practical purposes, the simple equation of J. A. C. Bresse, for which tables are available,[5] is sufficiently accurate. In using the Bresse equation the selection of constants is important. The value of the normal depth should be determined by the Manning formula, or better yet, by direct measurement. The value of C should be that which will make the approximation for the friction slope used in the Bresse formula most nearly correct according to the Manning formula, or

$$C = \frac{1.486}{n} \frac{R^{\frac{1}{2}}}{y^{\frac{1}{2}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

[26] "An Experimental Study of Backwater Curves," by Hsu-Hua Hu, thesis presented to the Graduate College of the State University of Iowa at Iowa City, Iowa, in 1947.

[5] "Hydraulics of Steady Flow in Open Channels," by S. M. Woodward and C. J. Posey, John Wiley & Sons, Inc., New York, N. Y., 1941.

Values of C should be so computed for the depths at each end of the backwater curve. Ordinarily such values of C will differ but little. An intermediate value is then selected for use in the Bresse formula.

Fig. 13 shows a comparision of curves computed by various methods with data selected from the experiments[21] by Messrs. Lansford and Mitchell. The upper set of curves approximates data shown as plotted points for a discharge of 39.98 cu ft per sec in a rectangular channel 5 ft wide, having a bottom slope of 0.003. The sides and bottom of the channel were covered with chain link, 2-in. diamond mesh, No. 9 wire. The normal depth was given as 2.335 ft, corresponding to a Manning's n of 0.027. The solid curve is that obtained by Messrs. Lansford and Mitchell using the step method with friction evaluated by the experimental rating curve with the assumption that the friction slope at a given depth varies with the square of the velocity. The dashed line below the solid curve shows the results obtained by using Bresse's equation with a value of $C = 39$, intermediate between 36.6 and 40.8. The dashed curve above the solid curve shows the results obtained by the author's method, with $m = 3.0$ and the hydraulic exponent n determined from Fig. 2 as 2.4. The agreement might be improved if the value of n were to be based on the experimentally determined rating curve.

As shown in the lower set of curves, similar results are obtained for a discharge of 29.91 with normal depth of 1.877 in the same channel at the same slope.

The author is to be commended for his scholarly paper. His succinct conclusions deserve thoughtful reading.

M. E. Von Seggern,[27] Assoc. M. ASCE.—Some important information regarding nonuniform flow in general has been brought out in the discussion of this paper, and the pros and cons of the writer's integration method have been argued. In this closure the various differences will be elaborated upon.

Mr. Posey raises an important point when he questions the underlying theory behind all present methods of computing backwater curves. The mere fact that, for many years, engineers and hydraulicians have accepted certain assumptions in regard to nonuniform flow is by no means proof that these assumptions are correct. More research on this subject is in order.

Mr. Kolupaila's suggestion of searching for a possible simple relation between the two hydraulic exponents has considerable merit. When Manning's formula is used in the computation of n, it does seem possible that, because both n and m can be reduced to functions of the cross section, a relation between these two exponents could be found. However, the writer has been unable to discover such a relation in a simple form.

Although the reader may have received the impression that Manning's formula is essential to the writer's method, this is not true. Kutter's formula, the Bazin formula, or some other open channel formula may be used; but, if any of these formulas is employed, then Fig. 2 is no longer pertinent, even though the principle behind it still applies.

[27] Engr., U. S. Bureau of Reclamation, Huron, S. Dak.

Mr. Kirpich has devised a convenient chart (Fig. 10) for determining the value of the hydraulic exponent, n, for any given depth, y. The integration of the equation of nonuniform flow as performed by Boris A. Bakhmeteff, Hon. M. ASCE, and by the writer is based on the assumption that log K plotted against log y is a straight line and can be expressed mathematically by the equation:

$$\left(\frac{K_1}{K_2}\right)^2 = \left(\frac{y_1}{y_2}\right)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (57)$$

which is an approximation only.

In a practical application it is necessary to obtain the value of the slope, $n/2$, of the straight line which most nearly approximates the curved line of the actual condition. Using Fig. 2, this average is obtained directly; and the degree of the approximation is apparent, as has been aptly stated by Mr. Posey. Using Fig. 10, the value of n for the extremes of depth would be averaged arithmetically. The choice of methods is a matter of individual preference.

Mr. Posey has questioned the reason for using the range of depths from 2.98 to 8.00 in Example 1 for determining the values of m and n. This question brings up a point that may have been overlooked by most readers. The writer's equation of nonuniform flow as expressed in Eq. 6 assumes a straight line plot of log y against log M over the range of depths from y_c to y. It is necessary, therefore, to use this range in computing the value of m. For the value of n it may be sufficient to use the shorter range from y_0 to y, but in all the examples the writer used the same range of depths for determining n as for determining m.

The writer is in complete agreement with Mr. Mitchell's discussion. It was, in fact, while attempting to simplify and shorten the work involved in computing the kinetic energy factor of Mr. Bakhmeteff's equation that the writer developed his method. Mr. Mitchell has not emphasized the one important difference between the Bakhmeteff method and the writer's method—the fact that the writer's method integrates both members of the equation whereas the Bakhmeteff method integrates only that part of the equation representing the effect of change in depth.

Mr. Mitchell suggests (and the writer agrees) that the effect of change in kinetic energy can often be neglected in practical engineering problems. According to Mr. Bakhmeteff this term can be safely disregarded only in the M_1-type of profile.[28] With the writer's method, however, there is no need to neglect it in any case, except, perhaps, for natural streams, as the entire computation is relatively short and simple.

Mr. Posey suggests that the equation of J. A. C. Bresse is sufficiently accurate for computing ordinary backwater profiles. The Bresse equation is based on a special case in which the hydraulic radius equals the depth of the water and would apply only to an infinitely wide rectangular channel, although it would be reasonably close for rectangular channels where the width is great compared to depth. However, if the hydraulic radius is assumed equal to the

[28] "Hydraulics of Open Channels," by Boris A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932, p. 98.

depth, then in the writer's method it can be shown that $n = 3.33$ (using Manning's formula) and $m = 3.0$. If a set of tables is abstracted from the writer's original set for these values of n and m, the necessity of computing the hydraulic exponents for each problem is eliminated, and the result is a method that should give exactly the same answers as the Bresse method—and one that may be even simpler to apply. Thus, some of the disadvantages of the Bresse equation are eliminated, and some of the advantages are retained.

Messrs. Hodges and Thatcher expressed doubt as to the applicability of this method to natural streams. The writer does not claim that the method applies to all cases, but it does to a large proportion of them. In natural streams the M_1-type profile is probably the most common. In this profile the effect of the change in kinetic energy is small and may be neglected. Thus, in Eq. 12 the Y-term can be eliminated so that $\Pi = Z$. Then Eq. 13 becomes

$$L = x_2 - x_1 = \frac{y_0}{S_0}(Z_2 - Z_1) \ldots\ldots\ldots\ldots\ldots\ldots (58)$$

In this form (Eq. 58) the critical depth does not need to be computed or the exponent, m, determined. Only the tables for the Z-function need to be consulted.

As has been stated by Messrs. Hodges and Thatcher, a natural channel has innumerable variations in both cross section and slope. The roughness coefficient also varies considerably from place to place. There are, in fact, so many variations that, except in rare instances, a "precise" computation is out of the question regardless of the method of computation employed. A rating curve at any point along the stream sums up and includes all these variables in the vicinity of that point; but even the rating curve cannot be considered 100% accurate.

If it is acknowledged that a "precise" computation is impossible, then the problem becomes one of deciding how much accuracy is required, and how much trouble and expense are warranted in obtaining data and making computations. For an integration method it will be necessary to know the average river bed slope, the normal depth, y_0, for any given discharge, and the value of the hydraulic exponent, n. If a rating curve or curves are available, $\log Q$ can be plotted against $\log y$ to obtain an approximately straight line the slope of which is $n/2$. Mr. Bakhmeteff shows one method of making this plot,[29] but the writer prefers the method shown by Horace W. King,[30] M. ASCE. The King logarithmic plot will give n and y_0. If the water spreads thinly over the flood plain, other complications arise, as discussed by Mr. Mitchell.

For backwater problems where data are meager and a preliminary solution is required for the early stages of planning studies, the integration method as presented herein or as given by Mr. Bakhmeteff (the two being nearly identical for natural streams) is particularly well adapted. A reasonably accurate profile can be obtained with a minimum of field data and a minimum of computation.

[29] "Hydraulics of Open Channels," by Boris A. Bakhmeteff, McGraw-Hill Book Co., Inc., New York, N. Y., 1932, p. 221.

[30] "Handbook of Hydraulics," by Horace W. King, McGraw-Hill Book Co., Inc., New York, N. Y., 1939.

Evèn where more detailed studies of the backwater profile are required, the integration method will give satisfactory results, considering the difficulties in estimating realistic values of the roughness coefficient and in selecting the proper location for cross sections. Aggradation of the stream bed by silt deposition upstream from the control structure which will produce the backwater profile may seriously affect that profile, and cannot be predicted in advance with any accuracy. So-called "precise" profile computations on a natural stream appear to the writer to be illusions in most cases; and, except where absolutely necessary, time and money spent on obtaining great volumes of detailed data and in making elaborate computations are unwarranted.

The step method of computation is not outmoded by any of the integration methods. As suggested by some of the discussers, many engineers who compute backwater profiles only occasionally will prefer one of the many step methods that have been published. In some cases it is not practical to apply an integration method.

For backwater profiles in uniform, prismatic channels, however, an integration method is simple, quick, and accurate. Some of the step methods published in textbooks and technical papers are no less complicated than is the integration method presented by the writer. Mr. Davison's method is a case in point. He begins with a form of the equation of nonuniform flow (Eq. 34*a*), which is the mathematical equivalent of Eqs. 1. By algebraic substitutions and computations he develops Eq. 40. It is doubtful whether most engineers who only occasionally compute backwater curves would find Eq. 40 any less formidable than Eqs. 12 and 13. The use of the writer's method does not necessarily require a complete understanding of the derivation of all the equations.

Where a complete water surface profile is not desired, but only one or two points are needed, the integration method is especially advantageous.

Mr. Hodges raises the question of applying this method to horizontal bottom grades. A few simple modifications in the equations are necessary for this extension, and the entire process is related to critical flow because y_0 approaches infinity as the bottom grade approaches zero. For a horizontal bottom grade; $S_0 = 0$, and Eq. 2*b* becomes: $S_s = -\frac{dy}{dx}$. For Eq. 7*a*, substitute

$$v = \frac{y_c}{y} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (59)$$

and let $Q^2 = K_c^2 \sigma$, in which σ is the critical slope or the slope necessary to make a given Q flow at critical depth.

After substituting in the equation of nonuniform flow and integrating,

$$\Pi = \frac{v^{-n-1}}{-n-1} - \frac{v^{m-n-1}}{m-n-1} \ldots\ldots\ldots\ldots\ldots\ldots (60a)$$

and

$$L = x_2 - x_1 = \frac{y_c}{\sigma}(\Pi_2 - \Pi_1) \ldots\ldots\ldots\ldots\ldots\ldots (60b)$$

Eqs. 60 correspond to Eqs. 12 and 13 for sustaining slopes. No tables are necessary. The application to a practical problem is the same as for sustaining slopes. Mr. Kirpich's charts for computing critical slope and critical depth are especially useful in these computations.

Mr. Escoffier offers an interesting development which results from a manipulation of the formulas presented in the paper. The deviation of the profile from a horizontal line in the instances he cites is small. The exponents, n and m, represent the slopes of lines which, in reality, are only approximately straight. For this reason, and because of Mr. Posey's pertinent query as to the soundness of the theory underlying the equation of nonuniform flow, the writer hesitates to comment on Mr. Escoffier's conclusion.

The writer is pleased at the response to this paper and wishes to thank all the discussers for their interest. It appears that the discussion provoked has been worthwhile.

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DISCUSSIONS

THE RELATION OF URBAN EXPRESSWAYS TO MASS TRANSPORTATION FACILITIES

Discussion

BY RALPH WESLEY JONES

RALPH WESLEY JONES,[7] M. ASCE.—The author opposes high-speed urban highways and favors the elimination in the downtown business areas of vehicular traffic (characterized as unnecessary), except mass transport vehicles (subways). Greater traffic volumes in cities do not occur in downtown areas because decentralization and by-pass routes have spread the load over greater areas. Mr. Bradfield claims that most of the older cities have an inconsistent street layout in central zones, which causes traffic congestion, and he recommends razing buildings and filling in harbor indentations to provide continuous routes.

The author's theme in opposing in-city expressways can be crystallized into almost one statement—that the traffic congestion at the central business (downtown) areas of major cities will be corrected if all but mass transport vehicles are ruled off the streets. As further improvement, he suggests that public transport vehicles be placed in subways. Obviously, if public and private vehicles disappear from the streets, there will be no vehicular street congestion; but vehicles do not disappear so readily.

The author states that people are wanted in the central business areas rather than private automobiles—and suggests that if the existing intolerable traffic conditions are deliberately permitted to be aggravated, such unwanted cars will stay away. He proposes that subways be operated in loop arrangement under all downtown business streets to serve offices, shops, theaters, and railroads—with transfer and "park and ride" privileges at terminals of streetcars and buses which would be located along this subway loop at the fringe of the central business area.

Several of the author's premises are open to question. He focuses all attention on traffic congestion at the downtown (central business) area, when traffic surveys clearly show that the greatest concentrations of city vehicular

NOTE.—This paper by Curtis G. Bradfield was published in December, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1949, by Elmer F. Steigelman, and Harold M. Lewis; and May, 1949, by Joseph Barnett.

[7] Cons. Engr., Philadelphia, Pa.

traffic are at points some distance from the center of the city. The author favors intercity expressways, but he opposes them within city limits for local motorists, and tolerates them only for through or cross-the-city traffic. Probably, one need is to elucidate what constitutes a "major city" and what constitutes a "local motorist."

At the 1940 census, there were only nine cities in the United States with populations exceeding 750,000; yet there were thirteen states with populations of less than 750,000. Surely, the author's statement that expressways are unnecessary inside cities, but are needed between cities is debatable. The area of New York, N. Y., covers 650 sq miles, which is half the size of Rhode Island, and one third that of Delaware. Local traffic must include that from metropolitan areas of cities; such areas increase the major city population by 50% or more, and the major city area by as much as 700%. In 1940, there were sixteen metropolitan areas with populations exceeding 750,000. The Philadelphia, Pa., metropolitan area is approximately the size of Rhode Island, so, if interurban expressways are proper for states like Rhode Island, they are more proper at larger cities.

The need for city expressways cannot be disposed of by empirical formulas, but must be determined by individual studies made at each community; nor should the author confine his planning to circumferential expressway belts as the only geometric pattern. Notable examples of satisfactory radial expressways include the East River Expressway and the West Side Highway in Manhattan (New York City) and the Parkway and Fairmount Park drives in Philadelphia. New York's Midtown Tunnel with approaches on Long Island is of incalculable value to this area. Expressways, like high-speed railways and railroads, also tend to enhance the values of certain adjacent benefited realty.

Municipal planners would agree with the author, that the motorist is not entitled to unrestricted parking and driving rights on city streets. A debatable point is that the local motorist should be legislated off the streets, but that the distant tourist should be welcomed for his shopping and hotel dollars. The local motorist is entitled to proper use—but not misuse—of the streets, because he has been assessed for their construction, and he is taxed annually to maintain them. Philadelphia has 33,000,000 sq yd of paved highways and 260,000 registered private automobiles—an average of 127 sq yd per car. What use would the author have the local resident make of such paving that he pays for? If he could not use it, why would he build it, or permit himself to be taxed for it?

It seems as questionable for the author to argue for in-city expressways for the convenience of the out-of-town user (to the exclusion of the local owner of the facility), as for one to suggest developing the United States for the convenience of some foreign nation. In modern highway planning, cities are bypassed for the many who prefer such routes, as well as for the benefit of the city itself; yet, expressways are constructed to connect conveniently the downtown commercial area with the outlying industrial and residential areas. In-city expressways at major cities cannot be justified on the basis of tourists alone. The present trend is for truck terminals to be located near principal through traffic routes, quite some distance from commercial areas in the center of the city.

Quite properly, the writer believes that motorists must not be given unrestricted parking or other traffic privileges; but, the solution to this problem is not too difficult. The author, however, bases his argument on the fact that 43 cars out of 90 cars parked in a central city parking lot were used by persons "going to employment"; such a survey means nothing in the over-all traffic problem which must catalog every vehicle entering or leaving the central area. Traffic problems would practically evaporate if all cars were anchored in parking lots instead of cruising about the streets. By actual statistics, 1,650 trolley cars in a major city made 33,000 trips daily—an average of 20 trips per car. It is probable, too, that some of these 43 drivers find a car essential to their employment, whereas others find public transportation from home extremely unsatisfactory. Origin and destination surveys show that the majority of persons driving to employment do so from necessity—not by choice.

The author's statement that expressways will preclude municipal development beyond the expressway (much as did the Great Wall of China), is not supportable. The fortification wall about Paris, France, could not permanently confine that city; nor could the Market Street Elevated Railway in Philadelphia restrict normal growth on either side, as the author contends. Actually, the reverse is true—the area tributary to this elevated railway developed phenomenally and there was a free interchange of traffic from one side of the "El" to the other, both by private vehicle and by mass transport lines.

The author claims that an expressway would save only from 7 min to 12 min from the suburbs to the downtown district, but that saving would not change the time of arrival at, or departure from, the downtown area. Naturally, the time saved would depend on the conditions prevailing—especially, the distance and the speed possible by both routes; however, with congestion so bad that 15 or 20 min are consumed sometimes, in driving 1 mile during peak traffic on regular streets at Philadelphia, an expressway would save much more than 7 to 12 min for persons living at its 9-mile edge. Expressways are more economical than conventional streets of equal carrying capacities, on the simple basis of possible velocity. The elementary formula for the carrying capacity per hour of a traffic lane is $\frac{\text{Velocity (feet per hour)}}{\text{Length of vehicle plus clearance between cars}}$. Thus, expressways save highway area. Additional economies of limited access expressways include fewer traffic officers, and less traffic control equipment—plus their added safety.

The points of greatest traffic volume at major cities are not at the downtown business centers—nor have they been for many years. Actually, in a city of 2,000,000 population, the total vehicular traffic at the maximum hour showed the principal downtown crossroads to clock only 40% and 45%, respectively, of the peak hour volume at arterial points 3 miles to the northwest and 4 miles to the north of the downtown point. On the principal highway, vehicular traffic volume increased progressively for $3\frac{1}{2}$ miles from the main business center—then it tapered off continuously. During evenings, every intersecting

street feeds additional vehicles into the principal artery of traffic, and also drains some off; in the mornings, the reverse is true.

This same main highway reflects other conditions that oppose the author's arguments against limited access highways. Thus, even in 1930, before today's aggravated conditions, the average vehicular speeds, inbound and outbound, were found to be as follows, for the distances shown from the center of the downtown business area:

Distance from center, in miles	Speed, in miles per hour
0–1	15
1–2	18
2–4	19.5
4–5	20.5
5–6	22
6–8	28

(Maximum traffic volume occurred at a speed of 19.5 miles per hr.) These statistics show that the volume of vehicles, alone, does not control the average speed; rather, a more important influence on speed is the number of intersecting streets where drivers must be "on edge" more constantly.

Although he agrees with the author that expressways will increase the need for parking facilities, the writer feels that such induced increase will be slight and that the large percentage of persons who conveniently can do so will prefer public transportation to driving their own cars; nor do the facts sustain the author's statement that major cities have failed to develop or modernize their public transport facilities in parallel with increasing use of the automobile for personal transportation. Mass transportation by private companies has proved so unsuccessful, financially, that the number of such operators has declined, by merger or otherwise, from 1,200 in 1922, to 478 in 1937. Still, in spite of such decline, total transit passengers gained 50%, from 15,600,000,000 in 1930, to 23,400,000,000 in 1946; in the same period, private automobile registrations, which are some measure of car usage, increased only 28%.

In 1920, the United States had in operation 800 miles of subway and elevated lines; by 1935, this had increased to 1,230 miles. Since the 1920's, the City of Philadelphia has expended approximately $170,000,000 for municipally-owned high-speed transit facilities; New York City has added greatly to its high-speed system. Much additional mileage would have been built had it not been for a long economic depression, followed by a long war that precluded construction. In some major cities, the local governments have stepped in with transit improvements when bankruptcy faced the private corporations.

The author makes a point that expressways do not prevent the healthy tendency to decentralize. Actually, expressways accelerate decentralization. The fact is that decentralization has been underway, progressively, in many cities, since long before the impact of the motor vehicle. Then, too, as cities expanded, so did business, industry, and homes over larger areas; yet, population statistics of urban places show clearly the absolute need for greater de-

centralization as follows:

Census year	Population (millions)	Percentage of total
1900	30	39.5
1910	42	45.7
1920	54	51.2
1930	69	56.2
1940	74	56.5

No one should expect to crowd the commercial, industrial, and social activities of such tremendously increased populations into the downtown central city areas as they existed at the turn of the century. Planners dare not attempt to restrict changes in modes of living that improve conditions. Workers, today, travel daily 20 to 25 miles to their jobs when, formerly, the majority of factory employees lived within walking distance of their occupations. Few workers who drive risk the conflict with city traffic, so, wherever possible, they use faster alternate routes.

Statistics show, too, that workers' traffic peaks occur at different hours for cities in different population groups. Industrial employees usually start and quit work as much as one hour earlier than office or store workers. Great numbers of industrial workers seldom see the downtown area, because they live and work in other parts. Decentralization is a proper trend, with community shopping and theater centers in surburban areas; expressways and other highway and transit facilities are made necessary to bind such sections together more effectively. Another traffic peak is found in most cities at about 9 p.m.—the entertainment peak. Surely, this evening vehicular traffic cannot be lured, in any appreciable volume, to the mass transportation lines. This traffic is profitable to general business; it will avoid unreasonably congested routes, but will make practical use of improved facilities, even if they be toll roads.

The author's statement that timid drivers and suburban matrons are afraid of expressways and will not use them is not on sound ground. Records of urban accidents show that 70% more collisions between motor vehicles occur at intersections than at points other than intersections. This is a striking point of argument in favor of limited access highways—with limited intersections. It is further support, and explains the reasons, for the tabulation made in an earlier paragraph, that the slowest urban vehicular speeds are realized nearer the city center although traffic volume here is not maximum. Obviously, the greater number of street intersections and the greater number of vehicles turning onto or off the main traffic artery, as well as pedestrians crossing (as they do near the city business center), slows up the traffic appreciably.

Records of rural areas showed that only one out of 11.6 accidents occured at intersections. Beyond doubt, street intersections are the principal scenes of urban motor accidents, and to eliminate such points of probable accident, by expressways or otherwise, is a step toward safety. The suburban matron on a shopping tour would find other statistics favoring her hours of travel—the

incidence of urban traffic accidents occurring in the following order of declining importance:

Severe periods of traffic accidents	Time
1	7 p.m.– 8 p.m.
2	8 p.m.– 9 p.m.
3	5 p.m.– 6 p.m.
4	12 p.m.– 1 a.m.
5	6 p.m.– 7 p.m.
6	9 p.m.–10 p.m.
7	11 p.m.–12 p.m.
8	10 p.m.–11 p.m.
9	4 p.m.– 5 p.m.

On-the-site observation, as well as study of street maps, reveals few larger American cities with the inconsistencies in their central city street layout that the author mentions, and for which he prescribes razing obstructions and filling in harbor indentations to make a more direct route. A rather remarkable rectangular pattern of streets, with certain radials, prevails at New York, Chicago, Ill., Philadelphia, Washington, D. C., Toronto, Ont., Canada, Seattle, Wash., San Francisco, Calif., Portland, Ore., New Orleans, La., Minneapolis, Minn., Saint Paul, Minn., Milwaukee, Wis., Los Angeles, Calif., Kansas City, Mo., Indianapolis, Ind., Detroit, Mich., Denver, Colo., and others, in spite of the fact that many of these face on curving lakes or rivers with the attendant difficulties to uniform layout. Few American cities can be called bad; and, here, it was nature who made certain requirements. For example, cities with difficult layout include Baltimore, Md., Buffalo, N. Y., Cleveland, Ohio, Pittsburgh, Pa., St. Louis, Mo., and (perhaps the worst) Boston, Mass. However, such European cities as Paris, Vienna, Austria, and London, England, where it is difficult to find a rectangular pattern downtown, may really be classed as bad. On the other hand, consider the age of these Eurpoean cities. As for filling in the harbor indentations, as is being done at Rio de Janeiro, Brazil, for a boulevard drive in a park section, this would be impossible at a port city with piers and docks jutting into the harbor. Also, such a filled in route across a harbor would provide a limited access highway, at least for the length of such cutoff, an improvement that the author opposes in favor of city streets with frequent intersections.

If the author has a complete remedy, by simply keeping vehicles out of the city and by putting mass transport underground, he does not need additional corrective measures. Actually, if mass transport were put below ground as he suggests, it could no longer suffer from the motorist; the latter could then blame no one but the motorist for any difficulties, if he had any. Actually, the automobilist would have no difficulty with traffic, if mass transport were off the streets. A winter strike of all operators of subway, elevated, trolley, bus, and taxicab lines at Philadelphia created a utopian condition for the motorist; he wished the strike would never end. This drastic step of taking mass

transport off the surface to aid traffic is not recommended; mass transport vehicles are troublesome, but necessary.

The author suggests more one-way streets, more traffic controls, and elimination of curb parking and truck servicing during business hours. Experience shows that most cities have approached the solution of these problems quite well. Philadelphia has 480 miles of one-way streets, nearly 22% of its 2,200 miles of paved streets. Probably, curb parking is the worst offender; and it is the easiest to correct, although the parker has certain rights, especially if he paid taxes for the street on which parking is restricted. Newer buildings are constructed to provide off-street truck handling, but one cannot legislate against the current practice; nor would one sensibly wish to do anything to harrass or destroy industry, the lifeblood of a community. Some relief to this problem is accomplished by pickup trucks, which retail loads between the various buildings and the motor truck terminals, thus reducing the number and size of vehicles on the downtown streets.

The author's statement that subways can be built at lower cost, and on a self-liquidating and profit-returning basis, will find little factual support. Municipal experts and transit operators are agreed that subways cannot be financially profitable. Local expressways could charge tolls similar to the successful practice with superhighways and bridges. Excellent arrangements include planning for joint development and use of a combined right of way for vehicular and rail traffic side by side. An old plan was to use the same bridge for a joint rail and highway crossing, with economy for both users. Some combined usage of limited access facilities by public and private conveyances is just as sane as is the practice of combining them on a city street or state highway.

Mr. Bradfield's statement that cities with densest centralization have the most extensive mass transport systems, and vice versa, is merely an observation of economics; sound railroading depends on revenue per mile, but mass transport cannot be built in sparsely settled sections with few riders per mile. Therefore, one can expect to find ample transit lines in densely populated territory, but one cannot expect to find satisfactory public transport where the area is thinly settled. As citizens tend toward the construction of homes with larger lots of ground away from the city centers, people will find an increasing tendency toward greater use of the private automobile, instead of travel by public utilities entirely, even though the private car may be used only to reach a convenient line of high-speed transit, or for shopping or relaxing at the neighborhood center.

Conclusion.—Cities must be planned today, as they always have been, for the foreseeable needs. Today, the planner can foresee farther than in former times. The earlier American colonial cities were built for the horse traffic of the day—even the horse car lines were not foreseen when many of these cities were planned. The trolley cars of the 1890's were not anticipated in the 1600's or the 1700's, or even in the 1850's, and no provision was made for them. Many downtown streets have never been widened, in spite of the fact that the city's population may have increased fivefold, tenfold, or twentyfold.

The greatness of a city depends upon the ease with which the people and their business can move from one part to another, and from any and all parts to the outer world. Expressways, and high-speed traffic arteries of various types, are as economically sound as is the turbine driven steamship of today in comparison with the sailing vessel of yesteryear. Actually, a given volume of traffic can be carried at lower expense—for construction, maintenance, and operation—on a limited access route than on an ordinary city street. Mass transport, for freight as well as for passengers, must be developed in cities, because merchandise is vital. Although, in any metropolitan area, it may be difficult always to justify each modernization or improvement on its own basis, engineers dare not obstruct progress. Economically, the old community hand pump cost less to operate than does municipal water supply, but people prefer the latter; they must bear the costs if they want the comforts.

In no city of 100,000 or more population can the public be served adequately by mass transport facilities, if the enterprise is to succeed financially. Private automobiles and taxicabs must supplement the streetcar or bus, especially for the off hours. Nevertheless, few cities of 100,000 population would think of expressways, although such cities are entitled to have through streets for faster movement of traffic, as well as by-pass routes. As for subways, if all the mass transport vehicles—that the author claims now move end to end on the surface of the many downtown streets—were to be placed in one or more subway loops, an impossible condition would arise.

Flow of traffic is very similar to flow of water—a common formula for the volume of either being, "flow equals cross section times velocity." Traffic flow, like the flow of water, is greater where the conduit is smooth, with few laterals to create side eddies. Much can be accomplished by observation of the simple principles of flowing water and applying these principles to the handling of traffic.

One fact is clear, when the various recommendations of the author are put together: If all vehicles (private and mass transport) are taken from the streets, expressways will be uneconomic; expressways will also be unnecessary—as, indeed, will be the streets.

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DISCUSSIONS

WIND STRESSES IN SEMI-RIGID CONNECTIONS OF STEEL FRAMEWORK

Discussion

BY C. BATHO

C. BATHO.[10]—The beam-line method for solving problems on frameworks having rigid, elastic, or nonelastic connections, which Mr. Sourochnikoff makes some use of in his paper, was first given in three papers in the Reports of the Steel Structures Research Committee (SSRC).[11,12,13] Since then it has been greatly extended, and, in particular, it has been applied to problems involving sway.

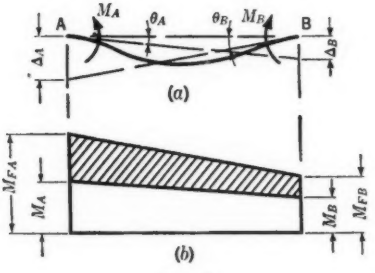

FIG. 7

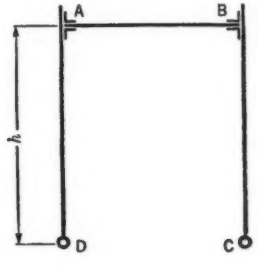

FIG. 8

The writer proposes to confine this discussion to a consideration of its application to a rectangular portal frame hinged at the feet, having nonelastic connections between the beam and the stanchions and subjected to vertical and side loads, since this is the main subject of Mr. Sourochnikoff's paper.

Consider a beam AB (Fig. 7(*a*)) loaded in any manner and partly restrained at the ends by moments M_A and M_B, and let θ_A and θ_B be the slopes at

NOTE.—This paper by Basil Sourochnikoff was published in February, 1949, *Proceedings*.

[10] Prof., Dept. of Civ. Eng., Univ. of Birmingham, Birmingham, England.

[11] "Investigations on Beam and Stanchion Connections," by C. Batho and H. C. Rowan, 2d Rept., Steel Structures Research Committee, Dept. of Scientific and Industrial Research of Great Britain, H. M. Stationery Office, London, England, 1934, p. 94.

[12] "The Analysis of the Moments in the Members of a Frame Having Rigid or Semi-Rigid Connections, Under Vertical Loads," by C. Batho and H. C. Rowan, *ibid.*, p. 177.

[13] "The Analysis and Design of Beams Under Given End Restraints," by C. Batho, Final Rept., Steel Structures Research Committee, Dept. of Scientific and Industrial Research of Great Britain, H. M. Stationery Office, London, England, 1936, p. 364.

the ends of the beam. If M_{FA} and M_{FB} are the moments necessary to reduce the slopes at the ends to zero, the beam may be regarded as having been initially restrained in this manner and the moments gradually lowered to M_A and M_B. The drop from B to the tangent to the beam at A(Δ_B) is therefore given by $\frac{A\,\bar{x}_B}{E\,I}$, in which A is the area of the shaded part of the diagram (Fig. 7(*b*)), $\bar{x}_B$ is the distance of its centroid from B, and I is the relevant second moment of area of the beam. Thus,

$$\theta_A = \frac{A\,\bar{x}_B}{E\,I\,l} \qquad (17a)$$

and

$$\theta_B = \frac{A\,\bar{x}_A}{E\,I\,l} \qquad (17b)$$

in which l is the length of the beam. This leads to the equations

$$6\,E\,K\,\theta_A = 2\,(M_{FA} - M_A) + (M_{FB} - M_B) \qquad (18a)$$

and

$$6\,E\,K\,\theta_B = 2\,(M_{FB} - M_B) + (M_{FA} - M_A) \qquad (18b)$$

in which K is I/l, the stiffness of the beam. By adding and subtracting Eqs. 18, two further equations may be obtained:

$$\frac{(M_A + M_B)}{2} = \frac{(M_{FA} + M_{FB})}{2} - 2\,E\,K\,\frac{(\theta_A + \theta_B)}{2} \qquad (19a)$$

and

$$(M_A - M_B) = (M_{FA} - M_{FB}) - 6\,E\,K\,(\theta_A - \theta_B) \qquad (19b)$$

Eq. 19*a* is the equation of a straight line connecting the mean of the bending moments at the ends and the mean of the slopes, and is called the mean beam line. Eq. 19*b* is the equation of another straight line connecting the difference of the moments at the ends and the difference of the slopes, and is called the difference beam line (see, subsequently, Fig. 10). They were first given in the Final Report of the SSRC.[13] If the restraining moments are equal and the loading symmetrical, they reduce to the one equation:

$$M = M_F - 2\,E\,K\,\theta \qquad (20)$$

which is the equation of the original beam line introduced to interpret the results of experiments on beam and column connections.[11] It is this beam line that Mr. Sourochnikoff has used in his example (see under the heading, "Analysis of Gravity and Wind Stresses in Connections Working Beyond the Proportional Limit: Symmetrical Rectangular Frame Under Symmetrical Vertical Load and a Side Load—Numerical Example").

Now consider the portal frame ABCD hinged at the base (Fig. 8). Let the relation between the moment, M, and the angular deformation, θ, of the connection (which is supposed to be known from experiment) be as shown in Fig. 9(*a*) for end A of the beam. The unloading line is assumed to be straight. If the columns were rigid this would give the relation between the slope at end A

of the beam, θ_A, and the restraining moment M_A. It will be called the restraint line of the connection. The attached member is elastic, however, and since it is pinned at the foot, the restraint line for this member will be

$$M = 3\,E\,K_{SA}\,\theta \qquad (21)$$

in which K_{SA} is the stiffness of the column. The resultant restraint line is obtained by adding together the abscissas of these restraint lines.

The restraint line for end B (Fig. 9(*b*)) is found in a similar manner. For the purpose of the following graphical constructions this line is reversed and drawn from the same origin as the restraint line for A (Fig. 9(*c*)).

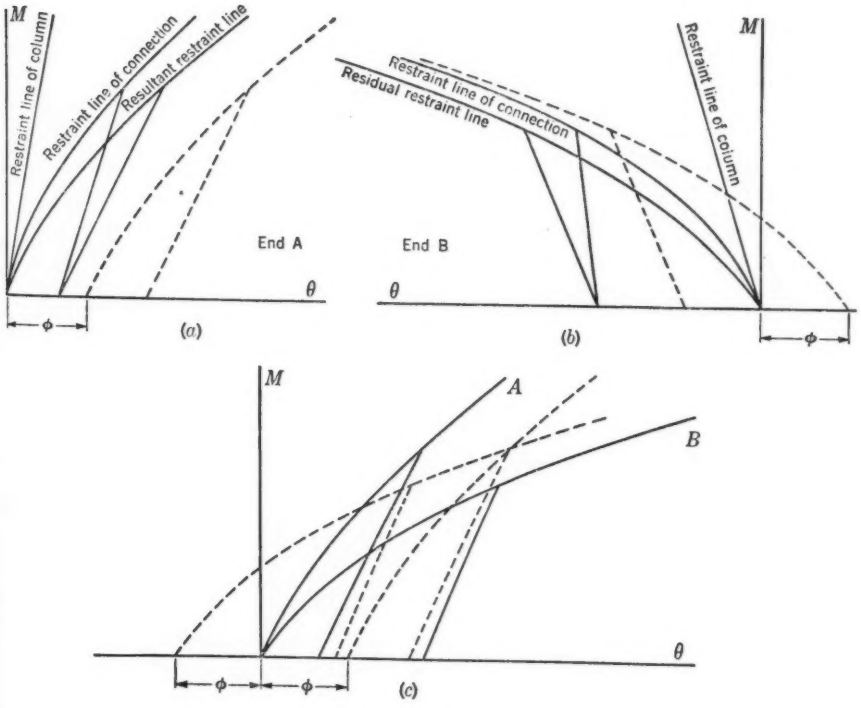

FIG. 9

If sidesway occurs, the horizontal movements at ends A and B must be equal and in the same direction, and, as the members AD and BC are of the same length, the angles of sway at A and B must be equal. Further, since the vertical members are pinned at ends C and D, sway does not produce any deformation of these members. Therefore the effect of sway may be represented by shifting the restraint line horizontally through a distance equal to the angle of sway (ϕ). For a sway to the right both restraint lines are shifted to the right, but, when the restraint line at end B is reversed, as in Fig. 9(*c*), this will appear as a shift to the left.

A particular example will now be considered (Fig. 10). The restraint lines will be assumed to be given by the curves A and B and the mean and difference beam lines for a given vertical loading of the beam to be as shown in the figure.

It is now necessary, as in the slope-deflection method and moment-distribution method, to determine the sway equation. If there is a horizontal force P acting at A it must be balanced by the horizontal forces produced by the moments at A and B. Therefore, with the convention of signs for the moment used in deriving the beam line, $M_A - M_B = P\,h$ if the load is to the left. If it is to the right, the right-hand side of the equation becomes $-P\,h$, and if there is no horizontal load it is zero.

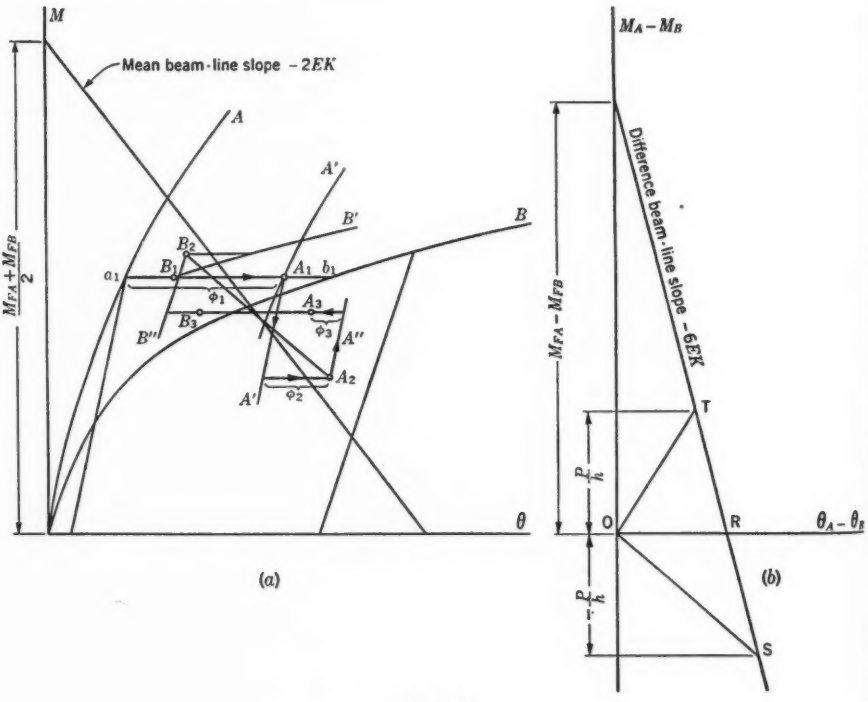

FIG. 10

It will be seen from the foregoing that the conditions to be fulfilled are:

(*a*) The shift of the restraint line for end A must be equal and opposite to that for end B;
(*b*) The sway equation must be satisfied;
(*c*) $M_A - M_B$ and $\theta_A - \theta_B$ must be in accordance with the difference beam line; and
(*d*) The mean of the moments M_A and M_B must lie on the mean beam line.

From condition (*c*) it follows that the relative position of the points on the ($M\,\theta$)-diagram (Fig. 10(*a*)) giving the moments at A and B must be the same

as given by the difference beam line. For example, if there is a horizontal load P to the left, the relevant points on the difference beam-line diagram are O and T. It follows that if a straight line is drawn between the points representing M_A and M_B in Fig. 10(a) it must be parallel and equal in length to the radius vector $\overline{OT}$ of the difference beam line. Further, from condition (d) it must be bisected by the mean beam line.

It is important to keep in mind that the end of the radius vector on the beam line is related to restraint line A and the end at the origin to restraint line B. The behavior of the frame under cycles of loading will now be considered.

1. *No Horizontal Load.*—Moment M_A must be equal to M_B. Determine the horizontal line $a_1 b_1$ on the ($M\ \theta$)-diagram bounded by the two restraint lines and bisected by the mean beam line. The relevant radius vector of the difference beam line is $\overline{OR}$. Set off half of this on each side of the mean beam line, obtaining points A_1B_1. These give the moments and the slopes at ends A and B of the beam. The horizontal distance from A_1 to the restraint line A is the angle of sway, ϕ_1, at A. Condition (a) is fulfilled since this is equal to the distance from B_1 to the restraint line B. The restraint lines now shift to A′B′.

2. *Horizontal Load to the Right.*—The quantity $M_A - M_B$ is now $-P\,h$ and the relevant radius vector of the difference beam line is $\overline{OS}$. Under this loading the moment M_A decreases; therefore, the unloading part of the restraint line A′ must be used. Determine the line A_2B_2 equal in length and parallel to OS, bisected by the mean beam line and having its ends equidistant horizontally from restraint lines A′ and B′. (This is best done by drawing the difference beam-line diagram on tracing paper and superimposing it on the ($M\ \theta$)-diagram.) Lines A_2 and B_2 give the moments and slopes at the ends of the beam, and the additional sway produced by the horizontal load is ϕ_2 as shown in the figure. The restraint lines are now A″B″. Only the unloading part of these lines is shown in the figure.

3. *Horizontal Load Removed.*—Moment M_A is once more equal to M_B. Determine the horizontal line bounded by the two unloading restraint lines and bisected by the mean beam line and proceed as for the case of no horizontal load (step 1). Lines A_3 and B_3 give the moments and slopes at the ends of the beam. The reduction in moment due to the application and removal of the side load is evident from the figure.

4. *Horizontal Load to the Left.*—

5. *Horizontal Load Removed.*—The method of procedure has now been explained and it is necessary to give only the results. These are shown in Fig. 11. One cycle of loading has now been completed and it will be seen that steps 4 and 5 result in a further slight reduction of the residual moments when the side load is removed.

Fig. 11 also shows the effect of succeeding cycles of loading. In order to avoid confusion the sways are only indicated for point A and some of the construction lines are omitted.

It will be seen that after the first cycle there is a slight change in the moments produced by the side load, but no further reduction in the moments when

the load is removed. The structure behaves elastically when the relevant restraint lines are the straight unloading lines.

The horizontal movements of the point of application, A, of the side load may be found by multiplying ϕ by the height of the frame, h.

If there are no vertical loads on the beam, or if the side load is large compared with the vertical loads, some of the moments become negative, and it is necessary to know the restraint lines for the connections under reversed moments.

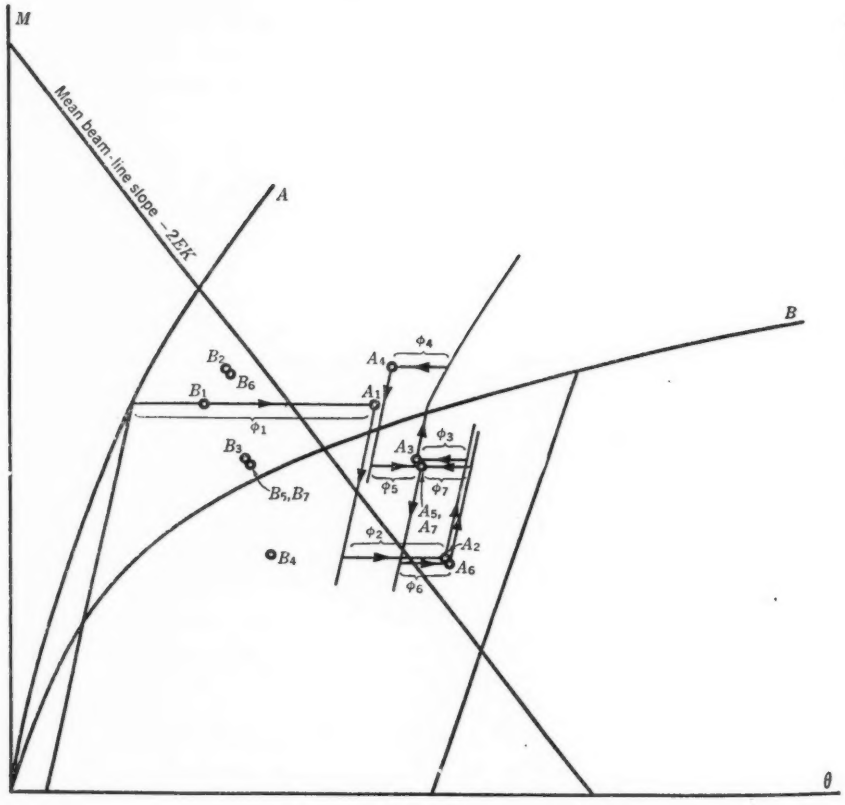

FIG. 11

Simplified Procedure.—The foregoing is the complete solution of the problem. If only the moments are required, a simpler procedure, suggested to the writer by A. J. Francis, may be adopted.

This procedure is shown in Fig. 12. The points a_1b_1 are obtained as in step 1, and the unloading line for end A, not allowing for sway, is drawn from a_1. Now draw two parallel lines 1 and 2 on tracing paper making the vertical distance between them equal to $P\,h$. Line 1 is to be used in conjunction with restraint line A, and line 2 with restraint line B. If the

horizontal load is to the right, line 2 must be above line 1. Also, draw a third line midway between these lines. Superimpose these lines on the (M θ)-diagram, keeping them parallel to the θ-axis and adjust them until the points a_2 c and b_2 are collinear. The ordinates of a_2 and b_2 give M_A and M_B. When the load is removed the residual moments are given by a_3 and b_3. The abscissas of the a-points and b-points do not give the slopes at the ends of the beam since sway has not been taken into account. The ordinates give the correct values of the moments because the sway shifts the restraint lines horizontally by equal and opposite amounts.

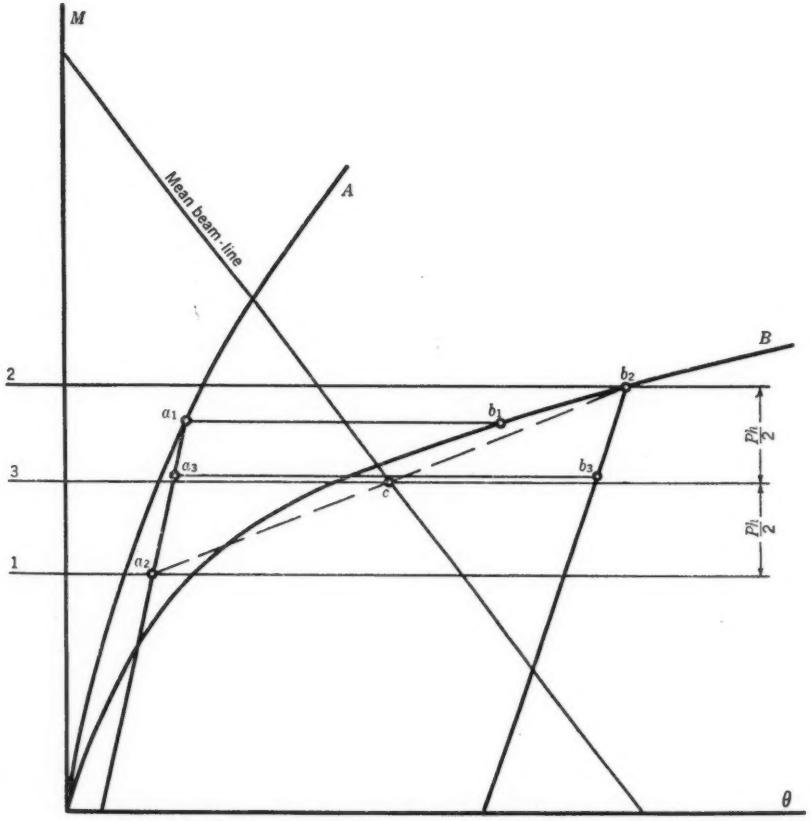

FIG. 12

In the example given by Mr. Sourochnikoff and shown in Fig. 3, it will be seen from the foregoing that the restraint line shown is really two coincident restraint lines superimposed, and the beam line ab becomes the mean beam line when sway occurs. By means of the foregoing simplified procedure, the moments may be determined in a simpler and quicker manner than that employed by Mr. Sourochnikoff. His results are substantially correct, but the

line between corresponding points such as A_4 and B_4 should be bisected by the beam line ab. Mr. Sourochnikoff begins by taking an arbitrary point B_1 and determines the side load corresponding to it. If the moments corresponding to a given side load were required, it would presumably be necessary to use a process of trial and error.

By means of the complete method given at the beginning of this discussion, which requires only a little more time than the simplified procedure, the sway—which may be of some importance—may also be determined.

It may be added that the reduction in the restraining moments at the ends of the beam produced by the application and removal of a side load was noted in the Final Report of the SSRC.[14]

[14] The Effect of Wind Loads on Frames with Semi-rigid Connections," J. F. Baker and E. Leader Williams, Final Rept., Steel Structures Research Committee, Dept. of Scientific and Industrial Research of Great Britain, H. M. Stationery Office, London, England 1936, p. 434.

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DISCUSSIONS

ANALYSIS OF PILE FOUNDATIONS WITH BATTER PILES

Discussion

BY J. OWEN LAKE

J. OWEN LAKE,[10] ASSOC. M. ASCE.—Because of the variable physical and mechanical properties of soils the actual forces induced in piled foundations cannot be predicted with any considerable degree of accuracy no matter how mathematically refined the analysis; yet, until a rigorous solution—even though based inevitably on certain simplifying assumptions—has been made, the degree of inaccuracy inherent in empirical expressions or formulas, based on assumptions which are obviously not realized in practice, can only be surmised. Therefore, Mr. Hrennikoff's analysis of the effect of fixity at the heads of pile. embedded in footings supported on nonparallel piles is of considerable values However, the particular example presented by the author exaggerates the importance of neglecting the lateral resistance of the piles, for not only are the piles spaced at exceptionally close centers but the passive resistance of the soil in front of the retaining wall is neglected.

Under the heading "Discussion of the Solution Results," paragraph *b*, the author states that the effect of the lateral pile resistances on the carrying capacity of the bent is small but that their effect on the lateral rigidity of the foundation is tremendous; for the particular example presented in the paper the lateral displacement is shown to be reduced six times by a lateral pile resistance which is less than 3% of the axial resistance. In view of this condition it must be observed that most foundations will have a deep capping to the piles capable of providing a resistance somewhat greater than the aforementioned 3%.

It should be observed that under certain conditions it is inadmissible to allow for any lateral resistance of the piles—for instance, where piles supporting relieving platforms to sheet pile bulkheads are located within the zone of expanding soil immediately behind the wall.

NOTE.—This paper by A. Hrennikoff was published in February, 1949, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1949, by Jacob Karol.

[10] Engr., The British Steel Piling Co., Ltd., London, England.

Under the heading, "Existing Methods of Analysis," the author states that the method developed by Swedish engineers,[3] for vertical piles loaded laterally, does not give a method for determining the unknown depth to the point of fixity. Another paper, however, by Donald Hamish Little,[11] describes an approximate graphical method, and an analytical solution of the problem is presented by the writer[12] in the written discussion of Mr. Little's paper.

[3] "Design of Pile Foundations," by C. P. Vetter, *Transactions*, ASCE, Vol. 104, 1939, p. 758.

[11] "Some Dolphin Designs," by Donald Hamish Little, *Journal*, Inst. of Civ. Engrs., November, 1946, p. 48.

[12] Discussion by J. Owen Lake of "Some Dolphin Designs," by Donald Hamish Little, in Supplement *ibid.*, October, 1947, p. 453.

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DISCUSSIONS

SOME EFFECTS OF ANAEROBIC DIGESTION ON SEWAGE SLUDGE

Discussion

BY A. L. GENTER

A. L. GENTER,[12] M. ASCE.—Sludge digestion undoubtedly diminishes the biochemical oxygen demand (B.O.D.) of sewage sludge. Messrs. Rawn and Candell question the reliability of B.O.D. determinations in measuring this reduction and the writer questions the reliability of the experimental method adopted in the paper to determine the B.O.D. reduction and to evaluate the effects of digesting sludge and returning it to primary effluent for common disposal by dilution in state waters.

In the series 1 laboratory tests the B.O.D. of the seeded mix dropped initially, subsequently rose to values definitely higher than those of the unseeded fresh sludge, and then continued to diminish gradually to some asymptotic low value. Under the experimental conditions used this sequence seems logical. During the active stage of anaerobic multiplication the B.O.D. should increase if the multiplying organisms have a B.O.D. of their own. Their continued multiplication is gradually arrested by the depletion of available food supply in the flasks and possibly by the simultaneous accumulation of decomposition products inimical to such growth in the sludge liquor itself.

The authors remark that this initial drop and subsequent rise and decline in B.O.D. values are not manifest in digestion tanks and state (under the heading, "Changes in B.O.D. During Digestion") that:

> "It is assumed that the conditions in the stage digestion tanks and in other functioning digestion tanks either forestall the rise of B.O.D. noted in series 1 or else mask it."

If this statement be true, and it evidently is, then such small bottle tests do not really simulate "* * * the conditions obtainable in the districts' digestion tanks * * *" as the authors previously claim under the heading, "General Description of Experiments."

NOTE.—This paper by A M Rawn and E. J. Candell was published in November, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1949, by W. E. Abbott, and Robert E. Leaver.

[12] Cons. Engr., Baltimore, Md.

The writer does not place much reliance on conclusions drawn from digesting sludges in 2-liter flasks. In relatively large digestion tanks operating in a treatment plant the biodynamic or biotic fostering activity is renewed daily, whereas in a relatively minute bottle testing system this is far from the case, even though the temperatures and detention times may be identical in both systems. In the actual plant system fresh sludge is added daily to the digesters so that its volume displaces somewhere from the digestion units the equivalent volume of digesting and digested materials in liquid condition. This procedure is not followed with tests conducted in 2-liter flasks where biochemical and chemical equilibriums become different as digestion progresses. The late W. L. Malcolm,[13] M. ASCE, minimized the effects of this possible experimental error in the digestion experiments conducted at Cornell University, in Ithaca, N. Y., in 1936.

However, whether or not bottle tests give a fairly relative indication of B.O.D. reduction through anaerobic digestion and whether or not more reliable methods can be developed for determining the B.O.D. of dilute and concentrated sludges, a closer examination of this method of disposing of sewage sludge reveals the fact that it levies a tax on primary clarifiers that operates in inverse proportion to their efficiency, especially when a plant is collecting sludges heavier and of higher B.O.D. than the sludge listed by the authors. Thus, practically, the better a primary sedimentation system functions in removing suspended sewage solids, the greater becomes the tax rate on the efficiency of the system.

This condition becomes quite evident in plain sedimentation treatment plants handling domestic sewage of about 300-ppm B.O.D. with sludge pumped to digestion containing as high as 9% solids. Here the combination of digested material with the plant effluent of 165-ppm B.O.D. could readily increase the latter by 7% for 30-day digestion and by almost 6% for 60-day digestion. A lowered clarifier efficiency may decrease this percentage but only at the cost of increasing the B.O.D. of the large fraction of plant effluent.

The authors in accounting for the removal of digester supernatant do not account for its disposal. The writer assumes that it is added back to the plant influent because it is dirty. This return is then accounted for in the 150-ppm B.O.D. of the primary effluent. In this case most of the digested solids returned to plain sedimentation return again to digestion to accumulate for 30 days and 60 days in that space to become a tax on its efficiency.

In any case one should not lose sight of the fact that digester liquor has a B.O.D. that can be double that of the plant influent and the B.O.D. of the wet digested sludge is almost a straight-line function of the amount of digested solids present plus the B.O.D. of the liquor present. Furthermore, even though 30-day digestion of a raw sludge of 9% solids content (as at Winnipeg, Man., Canada) may and does drop the sludge B.O.D. to 10% of its original value (in line with data presented in Table 3), this drop is from a far greater initial B.O.D. level and can leave the liquid digester contents of 5.6% solids with a total B.O.D. of 5,600 ppm, in place of the 1,200 ppm recorded for the digested

[13] "Studies in the Digestion of Ground Garbage," by W. L. Malcolm, *Sewage Works Journal*, May 1937, p. 389.

sludge having 2.33% solids in Tables 3 and 6. Increasing the ratio of sewage treated to liquid digester contents from 208 to 1 in the Rawn-Candell data to 490 to 1 in the case taken from the records of the Greater Winnipeg Sanitary District does not help as much as might be assumed. The penalty actually increases.

Thus, regardless of just how trustworthy the B.O.D. determinations may become, the writer believes that this highly telescoped method of sewage and sludge disposal can serve only to postpone the day of real reckoning. At best its penalty on plant efficiency lightens the taxpayers' burden by operating in some cases with just enough questionable adequacy under the guise of sewage treatment to satisfy the sanitary requirements of those residing above the point of discharge into state waters.

In reviewing various methods of sludge disposal the writer also believes that Mr. Rawn[14] presented a more convincing solution for this problem in 1941, than in the alternative solution now under discussion. In the former he gave excellent experimental evidence of increasing soil fertility assets by adding digested sludge to the soil where it is definitely needed. In the present contribution he and Mr. Candell offer a method of subtracting these fertility values from the soil and adding them to streams where their value is at least questionable.

[14] "Concerning Those Plus Fertility Values of Sewage Sludges," by A. M. Rawn, *Water Works & Sewerage*, April, 1941, p. 186.

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DISCUSSIONS

PROPERTY SURVEYS MUST FIT THEIR TITLES

Discussion

BY HENRY D. SOUTHERLAND, JR.

HENRY D. SOUTHERLAND, JR.,[8] ASSOC. M. ASCE.—The author is to be commended for his orderly presentation of the subject of real property title surveys to the engineering profession for discussion. The paper presents in a logical manner some of the basic principles of land title surveys, particularly as instructions for the general guidance of surveyors of land for title purposes.

The paper is somewhat provocative, however, in that it does not cover the solution to problems concerning real property surveys which do not fit their titles. The contents of the paper appear to deal with "making property surveys fit their titles" but from its title one would expect to find the reasons why, and conclusive proof that, "property surveys must fit their titles." Frequently the careful instructions of the author have not been followed and there are many title descriptions that do not adequately delineate the boundaries of the land intended to be described. It goes almost without saying that the property surveys "should" fit their titles, but by what sanctions "must" property surveys fit their titles? What is the consequence of a failure to conform with the author's instructions and what steps should one take to remedy the problems arising from inaccurate property descriptions? Reference is made to supplementary sources[9,10,11] as complete answers to these questions hardly could be given in a short paper.

It is to be regretted that the author has made several broad generalizations in the interests of brevity. Copious notes and citations would allow those persons who are interested to delve into the source material for a better understanding of the author's intended meaning and for additional information on the subject. For example, it is not probable that the correct definition of the

NOTE.—This paper by William C. Wattles was published in February, 1948, *Proceedings*. Discussion on this paper had appeared in *Proceedings*, as follows: May, 1949, by Robert F. Borg, and S. A. Bauer; and June, 1949, by Rupert F. Carroll.

[8] Birmingham, Ala.

[9] "Treatise on the Law of Surveying and Boundaries," by F. E. Clark, Bobbs-Merrill Co., New York, N. Y., 1939.

[10] "Boundary Problems and Development Problems," by D. G. W. Ricketts, Pelican Pub. Co., New Orleans, La., 1939.

[11] "The Surveyor and His Legal Equipment," by A. H. Holt, *Transactions*, ASCE., Vol. 99, 1934, p. 1155.

boundaries of a parcel of land should be so broad as to include "any other matters connected with the property in question." The author probably had reference to easements, encroachments, adverse possession, and any other matters connected with or affecting the legal title or the description of the boundaries, monuments, or area of the parcel of land in question, or of contiguous parcels. Although an accurate survey, by chance, may "be made without considering and weighing all" the items enumerated by the author, one cannot be assured of an accurate survey without considering and weighing these factors to the extent necessary, including "any other matters connected with the property in question" which may affect the accuracy of the survey.

The author refers to the relations between title and possession—that the surveyor must predicate his work first on the record and then on possession, and that the survey may be located by title of record disregarding the possessory claim just as if the possession were always a thing apart, distinct, and different from the title of record. The record title may be the evidence which delineates the property possessed. The color of title of record usually describes the land that goes along with the house occupied or the land used (Hicks versus Coleman, 25 Calif. 122, 85 Am. Dec. 103 (1864)).

The extent to which property surveys must fit their titles varies with the manner in which the result of such a survey is rendered and the purposes for which it is used. A verbal written description is the common method of using the results of a property survey and the penalty for errors therein varies with the purposes for which such title description is to be used and the parties involved. Property descriptions may be used in transfers by conveyance, transfers by will, in mortgages, in taxation, and for other purposes. The particular questions of controversy may involve adverse possession, statute of limitations, tax rolls, notice of sale for defaulted taxes, notice to third parties by recording, or other questions. The effects may vary depending upon the jurisdiction in which the land is located, upon local statutes, and upon the parties involved. For example, misdescription may not be fatal in showing valid taxation for 10 years and thus title by adverse possession (McCraw versus Lindsey, 209 Ala. 314, 95 So. 898 (1923)), but misdescription in the notice of sale for taxes would be fatal in a purported transfer of title at a tax sale (Craig versus Swader, 225 Ala. 366, 143 So. 553 (1932)). A description of real property for ad valorem tax purposes by reference to lot and block numbers of an unrecorded plat would be insufficient to give the court jurisdiction in rem in a tax foreclosure proceeding (Napier versus Runkel, 9 Wash. 2d 246, 114 P 2d 534, 137 ALR 175, 184 (1941)).

The author is very casual in dealing with the most important problem of the relative value of the property to the cost of making the survey by the simple statement that it is not considered in this paper; yet, the surveyor "must perform commensurate with the best practice." An "accurate boundary survey" readily may exceed the value of the property. One of the first considerations in any engineering survey should be the limits of accuracy and the permissible error of closure, which may be quite different in the survey of valuable city properties from those necessary in property surveys of inexpensive land located in rural areas.

One method by which the detailed work of the engineering profession is presented to the public is through land surveyors in the form of real property title descriptions. It would seem that the profession should make a stringent effort to have such work impeccable in order that the profession may build up esteem with the public through property owners, real estate agents, insurance agents, mortgage firms, taxing officials, courts, lawyers, and others who frequently come in contact with these detailed engineering descriptions which are filed as a matter of public record. Other detailed engineering work normally is handled by a restricted group of engineers, architects, contractors, or officials and does not come into contact with the general public except in the form of a completed project. The extent to which boundary disputes and inaccurate title descriptions still exist is an indication that such problems should receive greater attention. The author's paper should assist in accomplishing this objective.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

EFFECT OF DRIVING PILES INTO SOFT CLAY

Discussion

BY D. P. KRYNINE, AND ROBERT F. LEGGET

D. P. KRYNINE,[26] M. ASCE.—Subsidence of pile foundations driven through unconsolidated soft deposits is one of the most troublesome of engineering problems, particularly along the west coast of the United States. The question as to whether the pile driving itself may artificially convert a consolidated clay into a nonconsolidated clay and hence cause a drag on the piles with subsequent settlement of the structure is of vital interest. Hence the publication of the present paper is timely; and, although end bearing piles are considered, all conclusions of the paper as to the remolding of the clay by the piles are valid in the case of friction piles as well.

Support of a Friction Pile.—The well-known fact that, when a pile driven through clay is pulled out, there is generally some clay on it, suggests that the adhesion of the clay to the pile is greater than the shearing strength of the clay material. Thus, it may be safely admitted that a friction pile in a state of equilibrium is kept in place by the vertical component of the shearing force acting along the surface of the pile in its final position. The maximum of this component, or the bearing capacity of a friction pile, corresponds to the stage at which the average shearing stress at the surface of the pile equals the average shearing strength of the surrounding clay material (case of limit equilibrium). This condition of equality of the average shearing stress and the average shearing strength along a failure surface at the stage of limit equilibrium is a generally accepted assumption and as such it will be used in this discussion.

Stability of the Experimental Piles.—The writer uses the term "experimental piles" to designate the piles shown in Fig. 2. Consider the limit equilibrium of these piles 11 months after the pile driving, when the average shearing strength of the clay between El. 550 and El. 525 was roughly one half of the unconfined compressive strength (0.75 ton per sq ft). Assume that this value may be applied to the shearing strength of the soil material below El.

NOTE.—This paper by A. E. Cummings, G. O. Kerkhoff, and R. B. Peck was published in December, 1948, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1949 by Leonardo Zeevaert, Jacob Feld, Joseph C. Fegan, and Gregory P. Tschebotarioff; and June, 1949, by Philip C. Rutledge, A. W. Skempton, James R. Schuyler, and B. A. Kantey.

[26] Cons. Engr.; Visiting Prof. of Eng., Univ. of California, Berkeley and Los Angeles, Calif.

550 (in other words, assume that sand and gravel are replaced by weaker clay). The average circumference of a pile is about 3.3 ft and the length of a pile below El. 550 is about 60 ft. Therefore, the bearing capacity of an experimental pile, neglecting the support of the remolded clay above El. 550 and the bearing on the hardpan, would be $\frac{1}{2} \times 0.75 \times 3.3 \times 60 = 75$ tons. Certainly it is difficult to believe that this pile may settle under ordinary loading after the final shearing strength is acquired by the clay; and it may be stated that at the age of 11 months the experimental piles were substantially supported by the lower part of the clay deposit (below El. 550), and that in all probability very little pressure reached the hardpan (see Fig. 2).

Increase of the Bearing Power of the Experimental Piles Due to the Pouring of Footing.—The unconfined compressive strength, and hence the shearing strength of the lower part of the clay deposit (below El. 550), did not change very much after pile driving (boring P-1, Fig. 8) but increased considerably (boring P-11, Fig. 8) after the footing was poured (between 1-month and 11-month borings, as in Fig. 2). Thus, the increase in shearing strength of the clay, between 1-month and 11-month borings, is apparently a result of the application of the load to the pile—that is, a result of the increase of normal stress in the mass. Obviously, the shearing strength (boring P-11, Fig. 8) increased gradually, and immediately after pouring concrete the load was taken up by the pore pressure thus causing hydrostatic excess next to the pile and flow of moisture away from it.

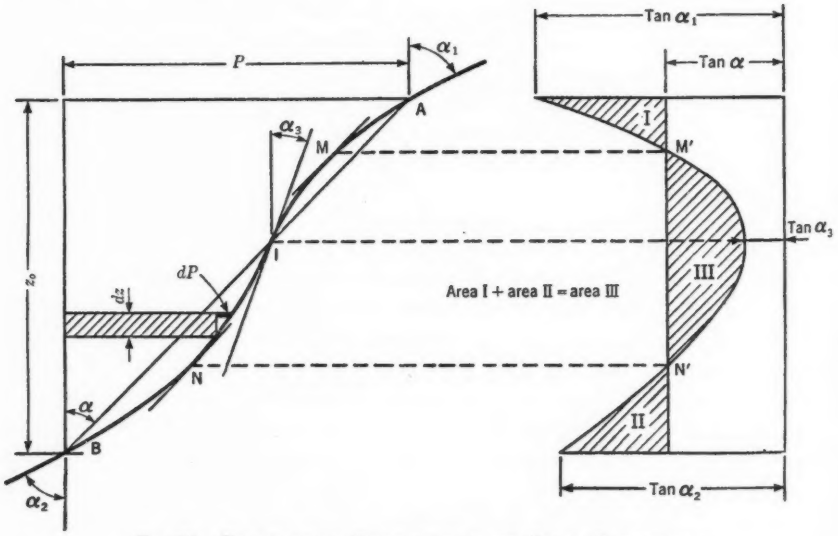

FIG. 24.—DIAGRAMMATIC COMPARISON OF THE LOADS CARRIED BY A FRICTION PILE AT DIFFERENT DEPTHS

Remolding of the Upper Part of the Clay Deposit.—The upper part of the clay deposit certainly was remolded, and the application of the load from the footing did not increase its shearing resistance appreciably—because much of the material was squeezed from the upper part of the clay deposit during the pile

driving, and there was not sufficient material to be compacted. At the lower part of the deposit, however, because of the higher shearing resistance of the material and its deep location, there was sufficient confined clay to be strongly compacted, with a corresponding increase in shearing strength.

Graphic Representation of the Bearing Power of a Friction Pile.—In June, 1948, L. Leroy Crandall, Assoc. M. ASCE, reported the results of a measurement of the bearing capacity of a Raymond concrete pile, cast in place in a corrugated steel shell, at Los Angeles, Calif.[27] At the center of the pile there was a steel pipe with SR-4 electric strain gages fixed at its surface. From the strains measured at different elevations and for different loads (and using the moduli of elasticity of the materials), the load carried by the pile was computed and represented graphically in the form of a family of curves, each curve corresponding to a load. Fig. 24 represents a similar curve in a simplified and idealized form. This curve may be analyzed as follows: Let P equal the total load carried by the friction pile; d, its diameter; and z, the depth measured from the top of the pile, the total length of the pile being z_o. Then the unit friction f (or shearing resistance of the soil at a given point) is

$$f = \frac{1}{\pi d} \frac{dP}{dz} \quad \ldots\ldots (4)$$

It is obvious that the derivative $\frac{dP}{dz}$ is tangent of the angle made by the tangent to the curve at a given point with the vertical. If the shearing resistance is constant along the entire length z_o of the pile, the curve in question would be the straight line AB making an angle α with the vertical.

In this case,

$$\frac{dP}{dz} = \frac{P}{z_o} = \tan \alpha \quad \ldots\ldots (5)$$

At the points where the tangent to the curve is parallel to line AB (points M and N) the shearing resistance of the material is

$$f = \frac{1}{\pi d} \frac{P}{z_o} = \frac{1}{\pi d} \tan \alpha \quad \ldots\ldots (6)$$

Finally, the minimum "friction" corresponds to the inflection point I of the curve; thus:

$$f_{\min} = \frac{1}{\pi d} \tan \alpha_3 \quad \ldots\ldots (7)$$

In Fig. 24, sum of the cross-hatched areas I and II is equal to area III. Figs. 25 and 26 represent such curves for the experimental piles between El. 565 and El. 525. Fig. 25 corresponds to the time before pile driving and Fig. 26 to a time 11 months after pile driving. In both cases the experimental piles have been considered friction piles. With the pile as a unit the influence of the remolding in this case is insignificant; but Fig. 26 should clearly show that the

[27] "Electrical Resistance Gauges for Determining the Transfer of Load from Driven Piling to Soil," by L. Leroy Crandall, *Proceedings*, 2d International Conference on Soil Mechanics and Foundation Eng., Rotterdam, 1948, Vol. IV, p. 122.

upper part of the clay deposit (above El. 550) contributed only slightly to the stability of the pile.

To reduce numerical computations, Figs. 25(*a*) and 26(*a*) have been prepared on the basis of simplified unconfined compressive strength diagrams.

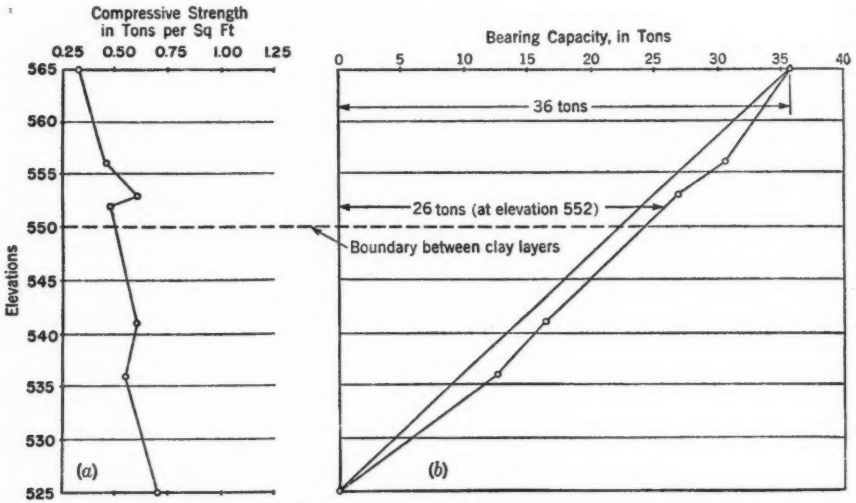

FIG. 25.—LOAD THAT WOULD BE SUPPORTED BY THE "EXPERIMENTAL PILES" BETWEEN EL. 565 AND EL. 525 IF THE CLAY WERE NOT REMOLDED

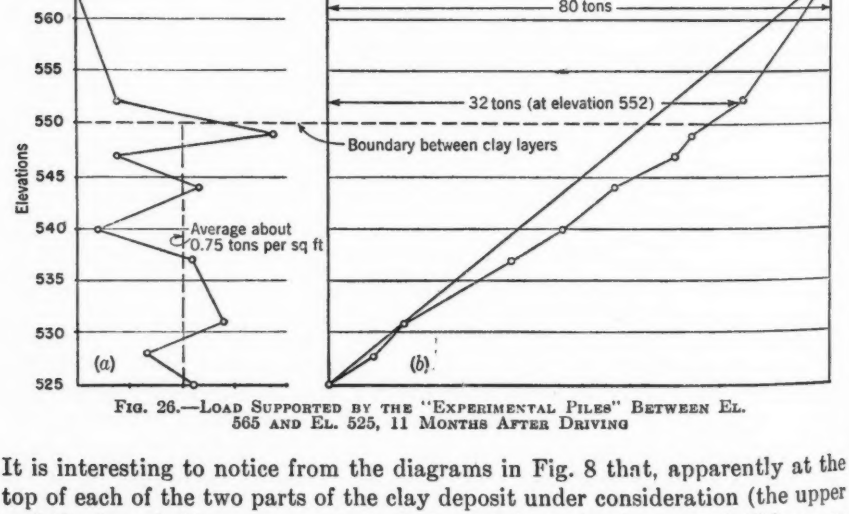

FIG. 26.—LOAD SUPPORTED BY THE "EXPERIMENTAL PILES" BETWEEN EL. 565 AND EL. 525, 11 MONTHS AFTER DRIVING

It is interesting to notice from the diagrams in Fig. 8 that, apparently at the top of each of the two parts of the clay deposit under consideration (the upper and the lower), there was a crust of about 5 ft, as in the case of Chicago or Boston clays.

Flow of Moisture.—Moisture in an earth mass moves under the influence of some difference of potentials—in this particular case, pore pressure difference. The flow problem in this case is quite analogous to that of the motion of a water drop on an oblique plane. The movement in the latter case occurs along the direction of the maximum slope, and in the former along the direction of the maximum hydraulic gradient offered to the moisture at a given point. At the extreme upper part of the clay deposit moisture probably moved vertically, whereas at deeper strata lateral motion from a pile (zone of large pore pressure) to the center of the cluster or toward the outside (zone of smaller pore pressure) was both a possibility and a probability.

Conclusions.—In this paper two different clay materials are considered—the upper and the lower parts of the clay deposit. The ratio of their thicknesses and their relative position made any considerable settlement impossible, although there was probably some insignificant consolidation of the upper part. If the total soil deposit consisted exclusively of the material in the upper part, there would have been remolding and consolidation of clay combined with settlement. However, if the total soil deposit consisted exclusively of the material in the lower part of the deposit, probably there would have been practically no settlement, except perhaps a brief period after pouring the footing.

A reader concludes from this paper that in a general case pile driving in clay may or may not cause consolidation and settlement, according to the properties of the clay. This conclusion does not contradict the opinion of A. Casagrande, M. ASCE. Nevertheless, the paper has destroyed forever the prejudice, very common among engineers, that the clay always loses much strength at points adjacent to the pile. As demonstrated by the paper, this loss of strength may or may not occur.

The paper would have furnished more topics for thinking if the water content of the upper part of the deposit (Fig. 9), and especially the void ratios of the material through the deposit before and after pile driving, were known to the reader. However, it is a valuable contribution to knowledge of clay properties, and the authors are to be commended for their fine work.

ROBERT F. LEGGET,[28] M. ASCE.—So many questions with regard to piles and pile driving remain unanswered that this paper is a most welcome addition to the literature of foundation engineering. The authors are to be congratulated on their initiative in arranging such useful field tests and in presenting their findings so concisely and clearly. The excellence of the presentation leaves the impression that the conclusions are logical and well founded. This was the writer's feeling after a first reading of the paper. Several subsequent rereadings, and discussion of the paper with colleagues, however, have raised such important questions that it seems only right to place these on record. The closing discussion should further enhance the value of the paper and may point the way for other investigators to develop still further the work which the authors have so well pioneered.

[28] Director, Div. of Bldg. Research, National Research Council, Ottawa, Canada.

Possibly the most important comment to be made is that all the conclusions of the authors are based on the interpretation of results from only one set of samples for each condition under review (that is, the P and C positions, 1 month and 11 months after driving). In this particular case, the lack of symmetry in the layout of the piles, the known variations in pile driving operations, and the fact that the problem is one in soil mechanics combine to make the absence of a duplicate set of bore holes unusually unfortunate. Knowing the care with which the sampling and testing must have been done, the writer would expect that duplicate samples would confirm the results obtained. In view of the importance of the results, however, such confirmation would have been very valuable particularly since all conclusions are based on the assumed accuracy of bole hole A as typical of the local soil conditions.

One interesting omission from the paper is any reference to the actual movement of soil which must accompany pile driving, with the exception of the statement (under the heading, "Introduction") that "* * * the ground surface between the piles heaved about 2 ft during the driving operations." A simple computation will show that this amount of heave accounts for about one half of the volume of soil actually displaced by the pile shells, allowing for a gradual tapering off of the heave outside the outer row of piles. As a consequence, some lateral flow of the clay, or some sudden escape of water, must have taken place. The direction, nature, and magnitude of this action would be dependent on the order in which the piles were driven. Such information is not given in the paper. Do the authors consider that the order of pile driving is of significance in relation to their studies? Would they relate their answer, whether positive or negative, to the lateral displacement of soil or to the release of water during driving?

The moisture contents of the clay as recorded by the authors are singularly interesting. (Detailed study, however, is hindered by the fact that the values for samples from hole A are plotted differently in Figs. 9(*a*) and 9(*b*).) The authors state (under the heading, "Results of Tests") that "* * * no trend can be discerned in the values of the water content in the soil at the center of a surrounding group of piles * * *." The data plotted in Fig. 9(*b*) seem to show some slight reduction in the soil moisture content after driving, thus emphasizing how valuable a duplicate set of records would be. Possibly the authors have more extensive records than those plotted, on which their comment was based.

The authors state (under the heading, "Interpretation of Results") that "* * * the water migrated horizontally from the disturbed zone near the pile to the much larger mass of relatively undisturbed clay between the piles." If this conclusion is correct, it should be possible to correlate the rate of flow presented by the authors' records with the known consolidation characteristics of the blue clay, since all the relevant factors are available. Have the authors investigated this correlation? It would be more than interesting to know if such an investigation confirms what the authors regard as the only explanation of the water loss.

The first borings taken after the pile driving are stated by the authors (under the heading, "Introduction") to have been put down 3 weeks after the

piles were driven. In view of the common experiences when piles are "set up" within a relatively few hours after having been driven, one is forced to the conclusion that soil moisture contents taken 3 weeks after driving do not give a complete picture of the soil disturbance in question. The "practical" explanation of "setup" is that water is released by the sudden disturbance of displacement of clay soil, during driving, and that it tends to lubricate the pile as long as driving continues. Subsequent readjustment of the disturbed clay, within the first day or two after driving, explains the usual difficulty encountered if redriving must be attempted.

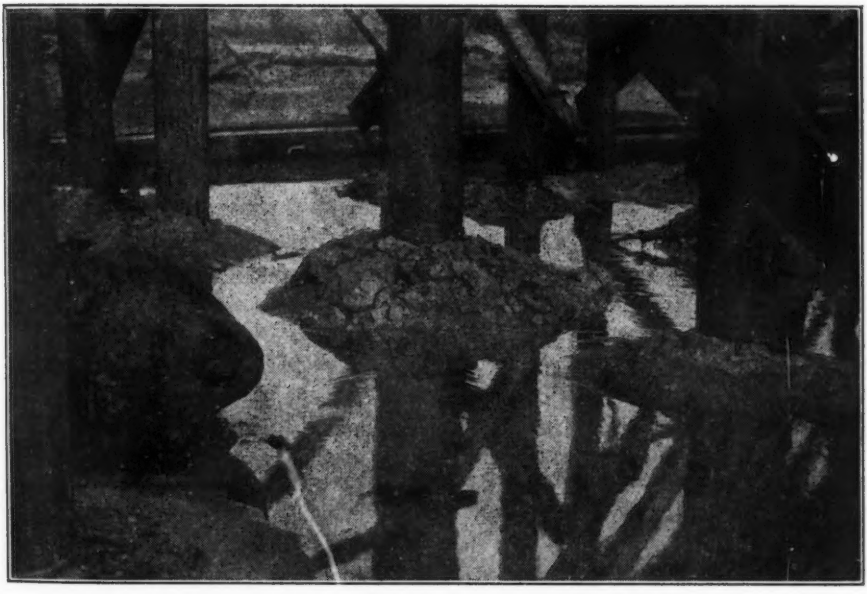

FIG. 27.—DISTURBANCE OF HARD CLAY BY PILE DRIVING

"Practical" explanations are often wrong but the writer has been able to observe the accuracy of this particular explanation in several locations. Fig. 27 shows some creosoted timber piles that were driven into hard Leda clay at a remote location on the north shore of the Gulf of Saint Lawrence, in Canada. Because of tidal conditions, it was possible to observe, at periods of low spring tides, the action of pile driving on the clay. Fig. 27 shows, clearly, how the ground heaved around the piles. The heaving around the pile on the right is typical of all that observed, the lumps of clay around the center pile having been caused by driving on to a small boulder buried just below the surface. In almost all cases, such as in the case of the pile on the right, in addition to the ground heaving to a distance about 2 ft from the pile, clay was squeezed up, in the form of a soft cream, around the pile in an annular ring not more than 1 in. wide. The clay is very stiff in its natural state so that in this case, at least, much of the water escaped vertically, an explanation which the authors discard

in the case they studied. Since the piles described in the paper were driven on dry land, it may be that the authors were able to watch the surface around the pile casings during driving; it would be interesting to know if this procedure were followed and, if so, what observations were made. Can the authors state explicitly that no water, or softened clay, escaped at the surface?

Accurate moisture contents are so essential a part of the investigation described that one is forced to question the statement (under the heading "Method of Sampling and Testing") to the effect that "A large part of the remolded sample was then used for the determination of the natural water content * * *." In view of this procedure, and the common laboratory experience with changing moisture contents, even when clays are handled in humid rooms, can this procedure be regarded as perfectly satisfactory? A further detail is the possibility of a change in moisture content between the two sides of the P samples. Since these samples were taken so very close to the pile shells, it is conceivable that a change in moisture content might have been observed between the sides of the samples adjacent to the casings and those opposite. Was any such difference observed?

Finally, the authors conclude by stating that their investigation

> "* * * does not warrant the conclusion that disturbance due to driving piles into soft clay is a source of detrimental decrease in the strength of these clays or a source of excessive settlement."

This statement is literally correct, of course, but it is almost inevitable that, in combination with the general title of the paper, this conclusion will tend to be applied to clays other than those from the Detroit (Mich.) area. Other studies may show similar results for other clay soils but until they do, and until some confirmation of the authors' results—even for the Detroit clay—has been obtained, it would seem advisable for the authors to be very much more specific in the defining of the rigid limitation of their conclusions. In all his experience, the writer has never encountered a clay soil with its natural moisture content so symmetrically fixed between its plastic and liquid limits. Many Canadian clays have natural moisture contents close to, if not exceeding, the liquid limit. The almost "ideal" character of the Detroit clay may possibly account for the findings of the authors and, if true, this fact would tend to limit the general applicability of the conclusions. However, it would in no way minimize the value of the findings in the paper to which the writer would pay full tribute. The very questions raised in this discussion show clearly the stimulation to critical thinking about the "pile problem" provided by this most interesting and valuable paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

MULTIPLE-PURPOSE RESERVOIRS
A SYMPOSIUM

Discussion

BY JOHN S. COTTON, AND ABRAHAM STREIFF

JOHN S. COTTON,[28] M. ASCE.—There is certain to be conflict between the various uses incorporated in the operation of a true multiple-purpose reservoir. The greatest conflict is usually between irrigation and power, unless reregulation is provided. There is less conflict between flood control and irrigation, or between flood control and power. The least conflict is probably between navigation releases and power.

There is no direct approach to the planning of multiple-purpose reservoirs. The best approach is to "cut and try"—that is, to assume various heights of dam and, for each height, to operate the reservoir taking into account the various requirements (to the degree possible) and the rule curves of operation. It is usual to find that it is not economical to provide for full flood protection, complete firm power delivery, or full irrigation releases in critical years. If more than 35 years of runoff records exist which show a reasonably definite and consistent flood flow pattern, it is not reasonable to provide definite flood control storage.

The next problem that arises is the yardstick to be used in arriving at the optimum size of reservoir. If the rate of annual return is used as an index, a very small reservoir will result. If the ratio of annual benefits to annual costs (annual benefits divided by annual costs) is used as an index, a larger reservoir will result. If the annual benefits minus annual costs is used as an index, a much larger reservoir will result. The writer is of the opinion that public projects should approach but not reach the limit set by annual benefits minus annual costs, because beyond this point the annual costs increase more rapidly than do the annual benefits. In the writer's opinion, this problem of basic economics, because of the very wide variations involved, should have as much or more attention than most of the other current problems concerning multiple-purpose projects.

NOTE.—This Symposium was published in March, 1949, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1949, by Herman A. Hilton, Wesley E. Gilbertson, Roger E. Amidon, L. C. Bishop, and A. H. Davison.

[28] Cons. Engr., San Anselmo, Calif.

In computing the annual benefits of a multiple-purpose project it is unnecessary to allocate the capital costs to the various uses. It is sufficient to lump all the benefits whatever they may be and to whomsoever they may accrue. It is logical to consider probable future increases in values of benefits and also to make allowance for indirect and intangible benefits. Great caution should be taken to avoid duplication of benefits, and to arrive at benefits that are truly practical and realistic. On the other hand, in all cases the annual costs, aside from operating and maintenance expenses, should include interest and depreciation (or amortization) charges.

In many instances private capital has provided some multiple-purpose benefits to the public through construction of projects developed for single purposes. It is unfortunate that some way has not been devised for the government to compensate private owners for the development of public benefits. If such compensation were possible, many private projects would have been built to optimum size instead of to some smaller size suitable for the private company's uses. This procedure would have obviated the underdevelopment of many fine sites, and would protect the future development of the relatively small remaining number of good sites.

The problem of reservoir silting is a serious one. Many ideas of silt control have been advanced, but nearly all will not meet the acid test of economic justification. It is true that proper farming practices will help the situation, but unfortunately the world's most important reservoirs are not in farm belts. The writer is aware of only a few rare cases in which excavating the deposits in the reservoir would be economically justified. Assuming that no money needs to be spent for storage of silt removed, it would cost at least $150 per acre-foot of silt excavated.

Ordinarily, the control of erosion on large steep areas by setting out plants, shrubs, and trees is not justified economically. The writer has examined vast areas so treated by the Japanese in Korea and Formosa. This work, especially in Korea, was done on an heroic scale. Many of the shrubs survived, but were dwarfed because the soil mantle had washed away. There is virtually nothing growing between the planted rows of shrubs and, as a consequence, the erosion is still severe. The writer has explored the Yellow River (China) drainage area. Nowhere in the world are erosion conditions so severe; and, at times, the Yellow River carries as much as 40% solids. The loessal area which provides the silt is 288,200 sq km in extent and the river drainage area is 771,500 sq km. It was found that this great erosion rate is caused by lack of vegetation, which in turn is very largely a result of overgrazing. A region was visited which the inhabitants had left 5 years before because of bandits. Surprisingly, a fine stand of grass and weeds was found even though the annual rainfall was only 6 in. The erosion from this area was practically nil. Some of the erosion came from very large deep gullies, but these could be stabilized by blasting rock canyon walls (where rock was present) to form rock-fill dams. In other cases, soil saving dams built of loess could be constructed to form barriers. Unfortunately the current war in China ended the program.

The writer has also observed the devastating effect of overgrazing in Mongolia, Ceylon, India, and Turkey. Unless carefully controlled, sheep do

much damage, but goats do much more damage. Therefore, on large unfarmed areas the economic approach to prevention of erosion is (1) controlling grazing very carefully, (2) spreading seeds in fertilized pellets from airplanes, (3) planting grass and shrubs in selected areas, (4) preventing fires, and (5) constructing cheap dams for gully control by blasting hillsides and by constructing soil saving dams. In short, let nature do most of the work.

Even if erosion is very severe and cannot be economically controlled, it is justifiable to construct a multiple-purpose dam, provided that the project can amortize itself during its lifetime. After the reservoir is filled with debris, run-of-river power can be obtained. By providing concrete pipes up the bottom of the reservoir and controlling them preferably from the downstream end, bed loads can be passed during the silting up period. These pipes could be used after the reservoir is silted up to drain off water from the debris. The concrete should be very porous, or should have holes in it and be covered with a rock and sand filter. The water yield would depend on the porosity and the size of the debris particles. It is obvious that this device would be suitable only for short reservoirs having relatively steep bottoms.

At times there is a tendency to install too much electrical production capacity at multiple-purpose projects. There is room for only one or two low-capacity-factor-power plants near the peak of a system load curve. Hence, other low-capacity-factor plants, due to insufficient or undependable energy production, cannot be fully utilized. Studies should be made to see if the firm power output, together with the firm output of existing and future plants, fits into the future integrated load curve of the system under all conditions. If, in addition, blocks of secondary energy can be produced, investigation should be made to determine if such energy is actually salable, or if it can be profitably made firm by steam support.

ABRAHAM STREIFF,[29] M. ASCE.—"Empty storage" (storage space kept empty in anticipation of floods) appears to be the most conflicting and least economical of multiple purposes for which reservoirs are used.

Until about 1910 flood control in the United States was obtained principally by the use of levees. In Ohio after the Dayton disaster in 1913 and the construction of the Miami project, flood control reservoirs in the upper reaches were introduced. Since then the flood control reservoir has descended into the plains combined with storage for other purposes. In the last Symposium paper, Mr. Hill has clearly set forth the existing conflicting requirements and has called for further discussion, which the writer undertakes, under five headings:

1. *Requirements of Flood Control.*—To prevent inundation a river channel must be of sufficient size to carry off maximum floods without overflowing its banks. Most of the rivers in the United States are still in a wholly unimproved native state, and are generally far too small to carry occasional great floods. As has been emphasized by Gerard H. Matthes, Hon. M. ASCE, J. C. Stevens, Past-President, ASCE, and others, river channels deteriorate rapidly when

[29] Cons Engr., Austin, Tex.

floods are absent. Vegetation chokes and blocks the channel, covering the sand bars. There is unrestricted encroachment on the river channel by riparian owners. Entire city sections are within the flood areas. The flood storage reservoir aims to reduce flood heights, but thereby creates only further neglect of and encroachment on the river channel.

The improved or "normalized" river, on the other hand, is maintained in an artificially corrected and protected channel, with increasing sectional area as it approaches the sea. The Rhine River in Europe is an example of such a "normalized" river. There are winter dikes, summer dikes, surface and submerged groins, longitudinal groins, bank revetments, maintenance dredging, and policing along its entire course. The Rhine River has ceased to overflow its normalized channel, and is at all times confined within that channel. "Empty storage" for flood control is not found along its entire course from Switzerland to the sea, and could not be tolerated on account of the great value of the land for other purposes. The Lower Mississippi is another example of a "normalized" river of very great magnitude and is virtually still under construction. "Empty storage" has been, and is being, provided on the tributaries of this stream.

2. *Value of Empty Storage in a Single Dam.*—At best, what is the value of an "empty storage" reservoir? There are four possible cases of flood damage:

(*a*) There may be flood damage to the uplands due to torrential rains and gullying, such as occurred near Taylor, Tex., in 1921. (Damage to the lands in such cases is enormous and wholly beyond protection by a reservoir.)

(*b*) There may be flood damage in the valley upstream from the reservoir.

(*c*) There may be damage in the valley downstream from the reservoir due to floods originating downstream from the reservoir.

(*d*) There may be damage downstream due to floods originating upstream from the reservoir.

Only in case (*d*) is the reservoir able (by reducing the flood height over a certain distance downstream from the dam) to protect the land areas between former and lowered flood heights. The "quick release" following storing of floodwater is generally not as simple a problem as might appear. For example, to release a million acre-feet in 10 days will require a release of 50,000 cu ft per sec for 10 days, which, in most large streams is a sizable flood in itself. Proponents of the method refer to a "bankfull" discharge as allowable. This tolerance, in an unimproved river, causes riparian owners, who watch their riverbanks being caved in by a boiling river under a sunny sky, to file damage suits immediately.

3. *Value of Empty Storage in a Chain of Dams.*—When the river has been transformed into a succession of reservoirs, however, the one protective function of four possible cases of flood damage cited under case (*b*) also disappears completely.

In such a system, like the Tennessee River below Chattanooga, Tenn., hundreds of square miles of the valley area below maximum flood lines were purchased over a length of some 630 river miles (nearly the straight line

distance between New York, N.Y., and Chicago, Ill.) and the entire population, including even the occupants of cemeteries, was moved out of the flood area. This system of flood "protection" thereby becomes identical to the levee system, where maximum flow lines are also the demarcation of the public domain which is reserved for the passage of the floodwaters, and it becomes the very system of the Mississippi River, of the Rhine, Seine, Po and Tisza rivers in Europe, and of many other rivers. As in the case of levees, such protection can be accomplished, of course, without building a single dam. By descending into the natural valley storage space, the "empty storage" reservoir becomes superfluous. At maximum flood heights (which are the most important) such "empty storage" reservoirs are just as uncontrollable as the original valley storage itself.

4. *The Dam Chain as a Normalized Channel.*—The present policy of federal agencies is to extend the Tennessee Valley Authority system to other watersheds. Leaving out of the discussion the storage dams themselves. the accompanying purchase and clearing of all the lands below maximum flow lines in itself, will serve an important purpose. The process of clearing these lands is equivalent to the establishment of a normalized channel, large enough to carry maximum floods, which is exactly what is needed. In other words, the levee system is extended far upstream into the main tributaries in a different form. The normalized channel now appears in the form of a series of reservoirs. Thereby the problem of maintaining a long, normalized channel is concentrated in the maintenance of a number of dams instead. This solution exactly fits the requirements of the United States, where a small population density makes maintenance of thousands of miles of normalized river channel uneconomical. Agricultural land area, on the other hand, is abundant. Where the levees begin in the lower stretches, the normalized channel must be maintained and dimensioned as before.

The maintenance of "empty storage" in dams that are completely contained within the valley storage is useless and does not merit repeating. Instead, the dams should (and could) be operated under full head for other multiple purposes. Carrying full storage will rout floods quicker; but, as long as the normalized channel is adapted to the requirements, the results will be satisfactory.

5. *Storage on the Watershed and in the Upper Reaches.*—For a dam containing storage space not only within but also above the valley storage, the conditions are as mentioned under case (*b*), for a single dam. Under such circumstances, improvement of flood storage efficiency can be achieved by a multiplicity of storage all over the watershed. For centuries this storage technique has been in existence in India. The province of Madras contains, on an area equal to that of New Mexico, 43,000 storage reservoirs with a total length of earth dike of 30,000 miles, containing some 200,000 masonry outlets. Runoff is stored during the wet monsoon season for use during the dry monsoon season. In this instance the storage is for irrigation purposes. When storage space is available in such quantity, the need for "empty storage," whether within or above the valley storage, is automatically eliminated and the stored waters can be allocated for all other multiple purposes without considering flood control.

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DISCUSSIONS

CONTROL OF THE HYDRAULIC JUMP BY SILLS

Discussion

BY EN-YUN HSU, AND STEPONAS KOLUPAILA

EN-YUN HSU.[9]—As has long been known, the hydraulic jump provides a useful method of dissipating the excess energy of supercritical flow and consequently of reducing bed erosion downstream from transition structures. Primary difficulties in designing such a structure are to insure the formation of a stable jump and to control its position for all probable operating conditions. Since various conditions may exist at different structures, different types of control are required. The authors have provided useful information for the control of the jump by using sills or abrupt rises in the channel floor for the condition that the downstream depth is smaller than the sequent depth for a normal jump. On the other hand, if the downstream depth is larger than the sequent depth for a normal jump, a drop in the channel floor must be used—as in the case for $h < 0$ mentioned by the authors. The problem of jump stabilization, by a drop, is encountered primarily at the end of a high-velocity expansion, and was studied by the writer at the Iowa Institute of Hydraulic Research, State Unitersity of Iowa, Iowa City, Iowa, under Project 67m of the Committee on Hydraulic Research of the ASCE Hydraulics Division. Since the results cover the incompleted half of Fig. 7, they are presented herewith.

Through dimensional analysis the general form of the relationship between V_1, d_1, g, d_3, and h may be indicated as follows:

$$\frac{h}{d_1} = \phi\left(\mathbf{F}, \frac{d_3}{d_1}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

in which V_1 (see Fig. 8) is the velocity of approaching flow; d_1 and d_3 are the depths before and after the jump, respectively; h is the magnitude of the rise (negative for a drop) in the channel bottom; and $\mathbf{F} = V_1 \sqrt{g\, d_1}$ is the Froude number of the approaching flow. As shown by the authors, the form of the function may be determined with good approximation for positive values of h through the use of the momentum and continuity principles. This procedure

NOTE.—This paper by John W. Forster and Raymond A. Skrinde was published in April, 1949, *Proceedings*.

[9] Research Associate, Iowa Inst. of Hydr. Research, State Univ. of Iowa, Iowa City, Iowa.

also holds true for negative values, except that several conditions of flow are possible.

For a given Froude number of the approaching flow, the downstream depth may fall in any of five regions. The lower limit of region 1 (Fig. 8(*a*)) is the depth at which the jump will begin to travel upstream and the upper limit of region 5 is the depth at which the jump will begin to travel downstream. Evidently the drop does not control the jump in these two regions. In fact, the jump, as such, is stable in position and the drop is effective for its desired purpose only in regions 2 and 4. The intermediate region 3 represents an undular state of flow without a breaking front.

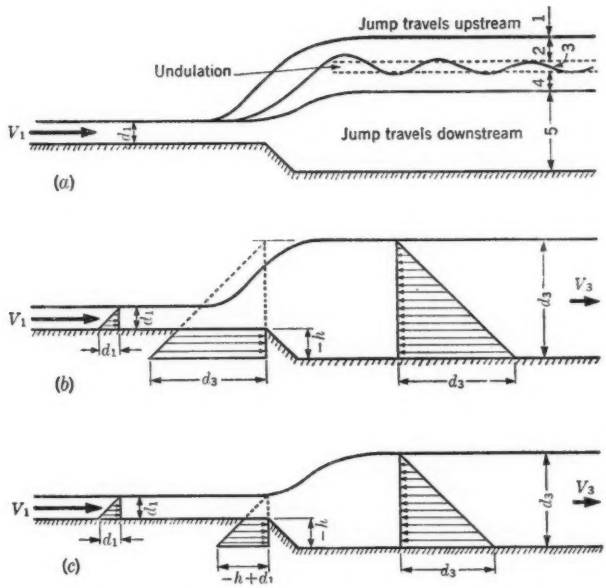

FIG. 8.—DIFFERENT FORMS OF THE HYDRAULIC JUMP AT AN ABRUPT DROP

To apply the momentum equation to either of the two stable conditions, an assumption must be made as to the longitudinal force acting on that part of the fluid in contact with the face of the drop. To simplify the analysis, the pressure on the drop was assumed to be hydrostatically distributed, its magnitude depending on whether the jump is upstream from the drop in region 2 or downstream from the drop in region 4. In other words, it is assumed that the intensity of pressure at the drop is proportional to the downstream depth in region 2, and to the upstream depth in region 4.

Based on these simplified assumptions, the momentum equation for sections (1) and (2) of region 2 (see Fig. 8(*b*)) is

$$\frac{\gamma d_1^2}{2} + \frac{\gamma [d_3 - (-h) + d_3](-h)}{2} - \frac{\gamma d_3^2}{2} = \frac{q\gamma}{g}(V_3 - V_1) \ldots . (21)$$

in which γ is the specific weight of the fluid and q is the discharge per unit width of the channel. From the continuity equation,

$$q = V_1 d_1 = V_3 d_3 \quad \ldots\ldots (22)$$

If Eqs. 21 and 22 are combined and reorganized, the following expression is obtained for region 2:

$$F^2 = \frac{1}{2} \frac{d_3/d_1}{1 - d_3/d_1} \left\{ 1 - \left[\frac{d_3}{d_1} - \left(\frac{-h}{d_1} \right) \right]^2 \right\} \quad \ldots\ldots (23)$$

In region 4 (Fig. 8(*c*)), the momentum equation has the form:

$$\frac{\gamma [d_1 + (-h)]^2}{2} - \frac{\gamma d_3^2}{2} = \frac{q\gamma}{g} (V_3 - V_1) \quad \ldots\ldots (24)$$

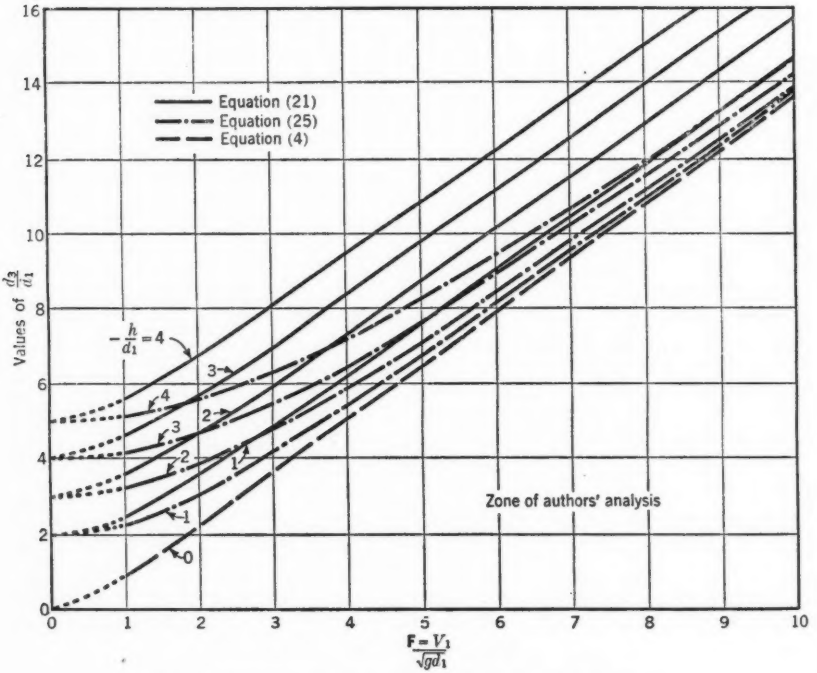

FIG. 9.—ANALYTICAL CHARACTERISTICS OF REGIONS 2 AND 4

If Eqs. 22 and 24 are combined, the counterpart of Eq. 23 is obtained for region 4:

$$F^2 = \frac{1}{2} \frac{d_3/d_1}{1 - d_3/d_1} \left\{ \left[\left(-\frac{h}{d_1} \right) + 1 \right]^2 - \left(\frac{d_3}{d_1} \right)^2 \right\} \quad \ldots\ldots (25)$$

Plots of Eqs. 21 and 25 are shown in Fig. 9 for various values of $-h/d_1$. As is to be expected, when $h/d_1 = 0$, both equations reduce to Eq. 4, which is the familiar expression for the hydraulic jump on a level floor.

Experiments to verify these relationships were conducted at the Iowa Institute of Hydraulic Research in the same 1-ft rectangular glass-walled flume used by the authors. The upstream end of the channel bottom was elevated by a 2-in. concrete slab, and an adjustable sharp-edged vertical sluice gate was used to regulate the upstream depth. The discharge was measured by a calibrated elbow meter. The highest Froude number obtained was approximately 10, and the h/d_1-ratio was varied systematically from 1 to 4.

Fig. 10 is the experimental counterpart of Fig. 9, only those parts of the analytical curves being retained which are followed by the measured data.

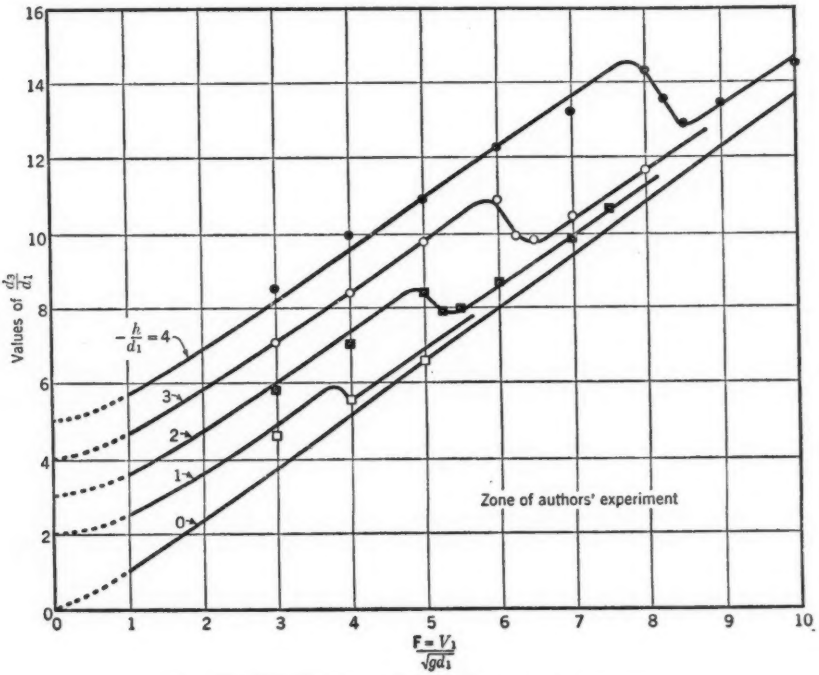

FIG. 10.—EXPERIMENTAL VERIFICATION OF THE ANALYSIS

The latter agree closely with the simplified analysis for both regions; and they also delineate a systematic trend of the undular transition between the two. The relative height of drop required to stabilize a jump for any particular combination of discharge, upstream depth, and downstream depth may evidently be determined from this diagram. Since the curves of Fig. 10 represent the missing part of Fig. 4, superposition of the two families on the same diagram should provide complete information for the control of the jump by either a drop or a rise in the channel floor.

STEPONAS KOLUPAILA,[10]—To estimate the sill head in a rectangular channel the authors use the formulas for the weir developed by H. Bazin in his investiga-

[10] Prof., College of Eng., Notre Dame Univ., Notre Dame, Ind.; formerly on the Faculty of Civ. Eng., Univ. of Vytautas the Great, Kaunas, Lithuania.

tions made from 1886 to 1895.[11] The propriety of applying these formulas for the jump conditions may be rather doubtful, because they are based on the assumption that the flow in the channel might be characterized by a comparatively regular distribution of velocities. The authors themselves ascertain this fact in their "comparison of results with theory." The application of several Bazin formulas thus has certain objections.

For example, the "theoretical" discharge formula, Eq. 10 (known as the Poleni formula of 1717) was never used without discharge coefficients, such as C_1 in the basic Bazin formula (for feet units)—

$$C_1 = 0.607 + \frac{0.0148}{H} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

In this case it is not necessary that H approach infinity.

This formula, like Eq. 12*a*, is also not limited by the condition $d_3 < (d_2 - \frac{3}{4}\,h)$. The condition (written by Bazin as $(d_2 - d_3) < 0.75\,h$) belongs to the third formula (Eq 12*b*) only.

The third formula for C_3 (Eq. 12*b*) is not the expression for the coefficient of Ph. Forchheimer; it is another of the Bazin formulas, which is clearly formulated even in the work of Forchheimer.[6] This correction for a non-aerated nappe is valid for $H > 0.4\,h$, but not for $H \leqq 0.4\,h$. It is believed that the latter is a mere typographical error. The writer was not able to check the statement of the authors that this condition was obtained during all experiments; therefore, it is not quite evident that the authors had applied the right formulas from the entire series proposed by Bazin. It may be that this fact could cause the big disagreement between the experimental data and the "theory."

In its experimental part, this paper is worthy of attention and can be a useful contribution in developing a practical application.

[11] "Recent Experiments on the Flow of Water Over Weirs," by H. Bazin, *Proceedings*, Engineers' Club of Philadelphia, Vol. 7, 1890, pp. 259–310; Vol. 9, 1892, pp. 231–244 and pp. 287–319; and Vol. 10, 1893 pp. 121–164.

[6] "Hydraulik," by Ph. Forchheimer, B. G. Teubner, Leipzig und Berlin, 1930, p. 382.

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DISCUSSIONS

APPLIED COLUMN THEORY

Discussion

BY WILLIAM R. OSGOOD, AND J. V. DU PLESSIS

WILLIAM R. OSGOOD,[25] M. ASCE.—One aspect of the art of engineering consists in designing without adequate information. However much one may admire the artist, most engineers, nevertheless, would like the art reduced to a science. In what he calls "the generalized Euler formula," and modestly suggests calling the "Euler-Engesser formula," Mr. Shanley has made a signal contribution to column theory—that is, to science. The formula, in the writer's opinion, should be called the Euler-Shanley formula. It is true that Engesser first proposed it, but Engesser later gave it up in favor of the reduced modulus, or double-modulus, formula. It remained for Mr. Shanley to show that Engesser was right the first time.

Mr. Shanley has traced nondimensional column curves back to 1929. Actually they go back further than that, to May, 1918, when R. W. Hawken[26] introduced them.

In connection with the determination of the constants K and n in the stress-strain formula, Eq. 4, the preferable way (certainly for a new material) of determining these constants is the following: Write Eq. 4 as

$$e - \frac{f}{E} = K\left(\frac{f}{E}\right)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

and take the logarithm of both sides:

$$\log\left(e - \frac{f}{E}\right) = n \log \frac{f}{E} + \log K \quad \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

Now plot the observed values of $\log\left(e - \frac{f}{E}\right)$ against $\log \frac{f}{E}$ and draw a straight line through the plotted points that seem to lie along a straight line.

NOTE.—This paper by F. R. Shanley was published in June, 1949, *Proceedings*.

[25] Senior Structural Engr. (Acting), David Taylor Model Basin, Washington, D. C.

[26] "Column Analysis and Design," by R. W. Hawken, Univ. of Queensland and the Sydney Univ. Eng. Soc., Brisbane, Australia, 1918.

The slope of this line will be n and the intercept on the axis of log $\left(e - \frac{f}{E}\right)$ will be log K. Alternatively, of course, $\left(e - \frac{f}{E}\right)$ may be plotted against f/E on log-log paper.

In connection with the interaction curves it may be well to emphasize that a bending moment M arising from transverse loads and an equal moment $P\,e$ arising from eccentrically applied longitudinal loads are not equivalent. If the interaction curve is obtained by transverse tests, as the curve in Fig. 13 was, then in using the curve for the design of an eccentrically loaded column, for a given e the load P read from the curve will be on the unsafe side—that is, the eccentrically loaded column will actually fail at a lower value of P. This becomes evident by considering Fig. 19. The moment diagrams on which Fig. 13 is based are like ABCD in Fig. 19, in which M is the maximum bending moment arising from the transverse loads. The corresponding diagram for terminal couples equal to M is AEBCGD. The maximum deflection produced by the terminal couples will be greater than that produced by the transverse loads. If an axial load is now added, it will take a smaller axial load to produce failure of the member loaded by the terminal couples than to produce failure of the transversely loaded member. (The terminal couples plus the axial load are the same as an eccentrically applied load.) On the other hand, if the interaction curve is obtained from tests on eccentrically loaded columns and the abscissas are $M = P\,e$, then in using the curve for the design of a beam-column for a given maximum moment from the transverse load the axial load P read from the curve will be conservative.

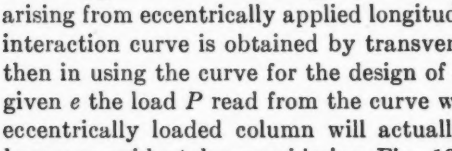

FIG. 19

In 1947 the writer suggested[27] a method of compromising between an unsafe design and an uneconomical one. The method assumes that the test data, in the form of an interaction curve, for example, are based on eccentrically loaded column tests. As intimated in the preceding paragraph, the eccentrically loaded column is a limiting case of a beam-column. The interaction curve for eccentric-load tests will be lower than the curve for any combination of axial and transverse loads, M being equal to $P\,e$ for the eccentric-load tests and equal to the maximum bending moment for transverse loads. As a practical matter it is easier to make accurate eccentrically loaded column tests than tests under combined axial and transverse load. The latter call for considerable art.

J. V. DU PLESSIS,[28] JUN. ASCE.—This comprehensive expression of the views of one who recently has shed further light on the fundamentals of column behavior is both refreshing and welcome.

[27] "Beam-Columns," by William R. Osgood, *Journal of the Aeronautical Sciences*, March, 1947, p. 167.
[28] Research Student, Faculty of Eng., Bristol Univ., Bristol, England.

It cannot be emphasized enough, however, that the "generalized Euler theory" is a "critical load" or "buckling load" theory and as such applies only to the "perfect" column or (the word used in this sense) "strut." Insisting, with the full rigor of Webster, that conditions existing are concentric loading, straightness, symmetricality, and homogeneity (even to postulating rigid monotonic rather than statistical location of atoms), it is possible to conceive a situation in which a column may be obtained in a straight loaded state, which, under some slight disturbance, will snap out of this position into the von Kármán nonstraight equilibrium bifurcation. In some cases, if the velocity of buckle is such as to induce large dynamic energy effects, the strut will surmount the barrier of increasing equilibrium loadings for larger deflections and then, behaving according to the negative slope of the deflection-end-load curve, rush to catastrophic collapse. If this last condition were in danger of occurring, the use of the buckling criterion would deserve some support as a logical design basis in order to guard against the occurrence of the "perfect" column. Since the column in practice, perfect or otherwise, seldom occurs in splendid isolation, as in some types of water towers, but generally with some measure of restraint afforded in adjoining framing, it is not reasonable to fear the "runaway" of any incidentally occurring perfect column. Apart from the laboratory difficulty of securing the perfect straight equilibrium bifurcation, even in the "short column" compression test, practice adequately indicates that column failure is a matter of bending rather than buckling.

To cite a practical case, an old reinforced concrete frame building was subjected by new owners to warehouse type loading in the region of two to three times the design loading, and one of the columns under the loaded floor began to fail. Distortion was a gradual process (was not suddenly there) and proceeded so slowly as to allow adequate time for supporting timbers to be introduced. Whatever it was that happened, it certainly was not buckling. Apart from such records of column failures as may exist, the laboratory struggle to produce perfect pin-ended columns is sufficient testimony that column failure in civil engineering structures is a matter of bending rather than buckling, and that columns undergo bending deformation from the inception of loading.

On the basis of this observation it is impossible to agree with the author's statement (in Section 1) that:

> "It is doubtful whether any structural engineer would wish to take advantage of this factor [increasing equilibrium load with increasing deflections] in actual design, because in this region an increase in stress will involve a large amount of permanent set. It seems logical, therefore, to regard the beginning of the bending [that is, buckling] in a perfect column as the upper limit for structural purposes."

As a basis of discussion, accept the premise that a certain amount of permanent set is the maximum permitted for design purposes. If bending occurs from the beginning, due to some cause classified by buckling theorists under the term "imperfection" (as it undoubtedly will), the critical amount of permanent set will be induced at a lower load than in the case of the initially

perfect but later buckling strut. The permissible load will therefore be different in practice from that derived from a buckling analysis. For other criterions of design, similar arguments will still apply. The greatest argument in support of buckling formulas is that there is no other theoretical analysis that does not presuppose knowledge either unobtainable or securable only by investigation or test, and, in any case, whose degree of variation has never been subject to control or even adequate determination. Until an adequate theoretical analysis not requiring unknown information exists, and until determined and specifiable systems of tolerances are used, it appears more practical to use empirical formulas and data, or practically proved formulas, rather than to rely on a formula based on a case unlikely to occur in practice. The interaction curves described by the author are a valuable step in this direction; but they still involve undecided tolerances and, by so doing, only serve further to underline the need for investigation as to the range of tolerance that actually occurs in practice.

Although buckling formulas are more of academic interest, their value as one set of known boundary conditions to a complex region of structural actions should not be forgotten, and in this connection the author's clarification of the stress situation that may occur across a section is of the greatest value.

It is worthwhile to stress the fact, however, that this clarification (which, in effect, is a statement that the ratio of stress change to strain change applicable to analytical consideration of any fiber is, in some cases, for all the fibers, the ratio corresponding to the loading stress strain relationship) applies in some (and probably in most) instances to columns which for some reason (such as eccentric loading) bend right from the inception of the loading program. Any load analysis or distortion analysis of columns in the elasto-plastic range must therefore be subject to "co-analysis" as to whether all the fibers are being loaded or whether some are being unloaded. Analysis of the condition for loading to occur over the entire section has been made by D. C. Drucker,[29] Jun. ASCE, for the case of buckling of the perfect column, and extension can be made on similar lines to the more general "non-perfect" column problem. The case in which (given increasing end load) all fibers continue to load is obviously more tractable analytically since it is not necessary to have a stress-strain relation sensitive to "direction." In this category may also be placed those cases in which, if a fiber is subject to unloading, it is a fiber that has been subject to a past history of only elastic strain. Only the perfect, or near perfect, columns are likely to fall outside this category before reaching the maximum load they will carry; and, in general, this will be more likely to occur with materials having relatively steep-sloping strain-hardening curves. Therefore, the arguments that led the author to support the tangent modulus Euler formula are of much wider application.

[29] "The Plasticity of Metals," by D. C. Drucker, *Proceedings*, ASCE (publication pending).

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DISCUSSIONS

AN ENGINEERING CONCEPT OF FLOW IN PIPES

Discussion

BY E. L. HARRINGTON

E. L. HARRINGTON,[22] ASSOC. M. ASCE.—The Darcy equation is probably the best method for computing flow of liquids in pipes, but the Hazen-Williams formula, in spite of its fallacy, as expressed in the paper, is an excellent formula for computing flow of water in pipes. The Hazen-Williams formula is usually expressed as

$$V = 1.318\, R^{0.63}\, s^{0.54}\, C \quad \ldots\ldots (5)$$

The formula may also be expressed for flow in circular pipes in the following manner:

$$Q = 0.01037\, D^{2.63}\, H^{0.54}\, C \quad \ldots\ldots (6)$$

in which Q is the rate of flow in cubic feet per second; D is the diameter of the pipe in feet; H is the head loss in feet per 1,000 ft of pipe; and C is a coefficient. In this form Eq. 6 may be solved directly for Q if D, H, and C are known; or, it may be solved directly for D if Q, H, and C are known, which avoids the trial and error solution necessary in the Darcy equation.

An experiment on the flow of water in a pipe made on the water supply system of the Port of Manzanillo, Colima, Republic of Mexico, from which the following information was obtained, is of interest.[23] The basic data were as follows:

Description	Quantity
Length of Pipe—	
In kilometers	16.93
In miles	10.52
Diameter of Pipe—	
In meters	0.20
In feet	0.6562
Available Head—	
In meters	138.00
In feet	452.75
Rate of Discharge—	
In cubic meters per second	0.042
In cubic feet per second	1.483

NOTE.—This paper by Charles W. Harris was published in May, 1949, *Proceedings*.

[22] Associate Prof., Dept. of Civ. Eng., Agri. and Mech. College of Texas, College Station, Tex.

[23] "Hydraulica," by Pedro J. Dozal, Talleres Gráficos De La Nación, Mexico, 1938, p. 322.

The pipe was described as being of, "cast iron, of very smooth surface, and of uniform diameter." Using the given data—$V = 4.39$ ft per sec, $f = 0.0179$ by the Darcy equation, and $C = 140$ by the Hazen-Williams equation. As early as 1905 a coefficient of 140 was predicted for "very best cast iron pipe, laid perfectly straight, and when new" by Gardner S. Williams and Allen Hazen,[24] Members, ASCE. The experiment at Manzanillo, on a pipe approximately 10 miles long, indicates the accuracy of the predicted coefficient.

Flow of water in pipes may be computed by the use of the Hazen-Williams equation and checked by the Darcy equation using appropriate friction factors. The Hazen-Williams formula apparently gives good results and is popular among practicing engineers.

[24] "Hydraulic Tables," by Gardner S. Williams and Allen Hazen, John Wiley & Sons, Inc., New York, N. Y., 1st Ed., 1905, p. 3.